Dependence Graph Preprocessing for Faster Exact Modulo Scheduling in High-level Synthesis

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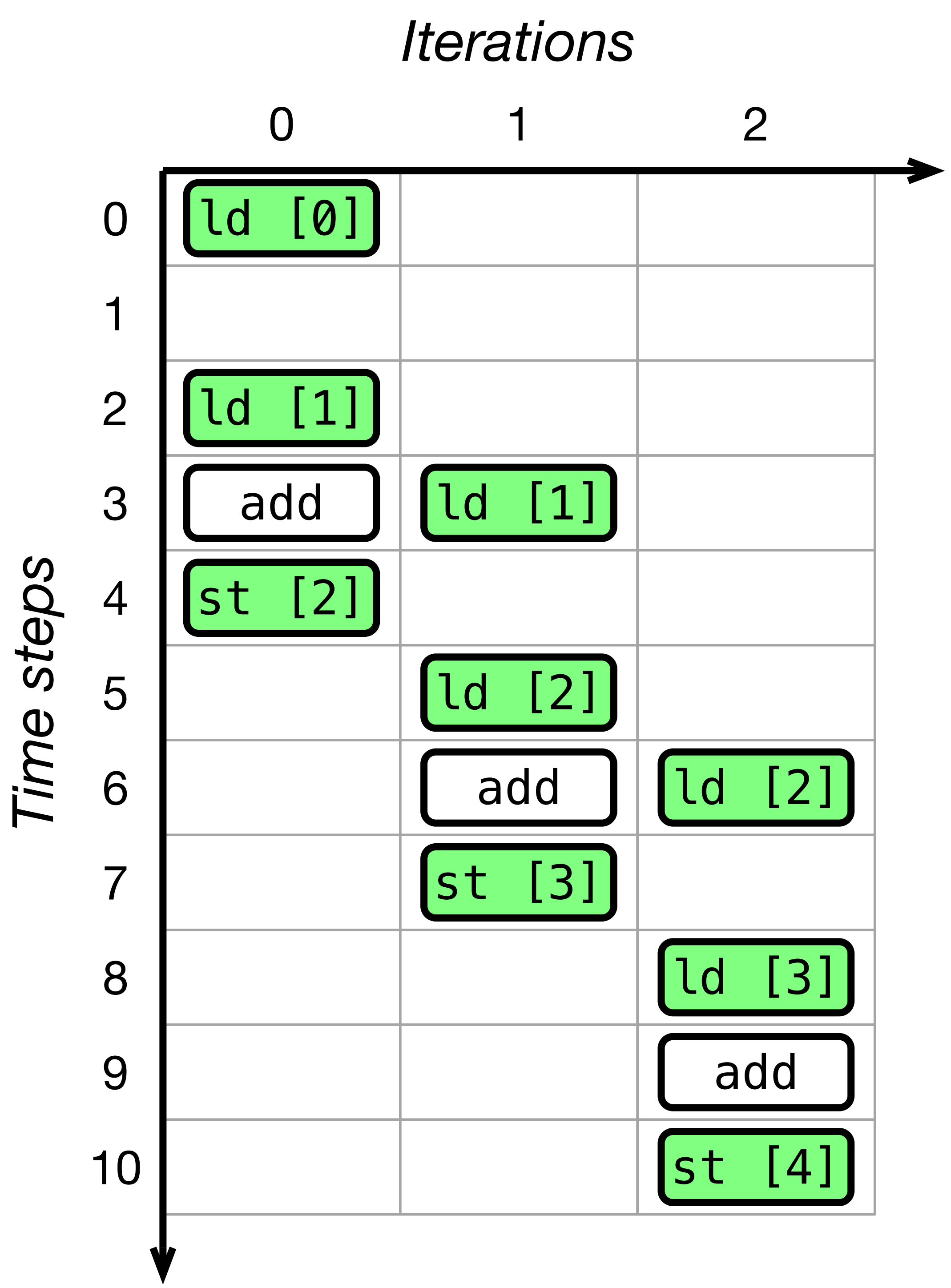
Agenda

- **(Short) introduction to modulo scheduling**
- Proposed preprocessing approach
- Results and insights

Modulo Scheduling

- **Loop pipelining**
  = increase throughput by overlapping iterations



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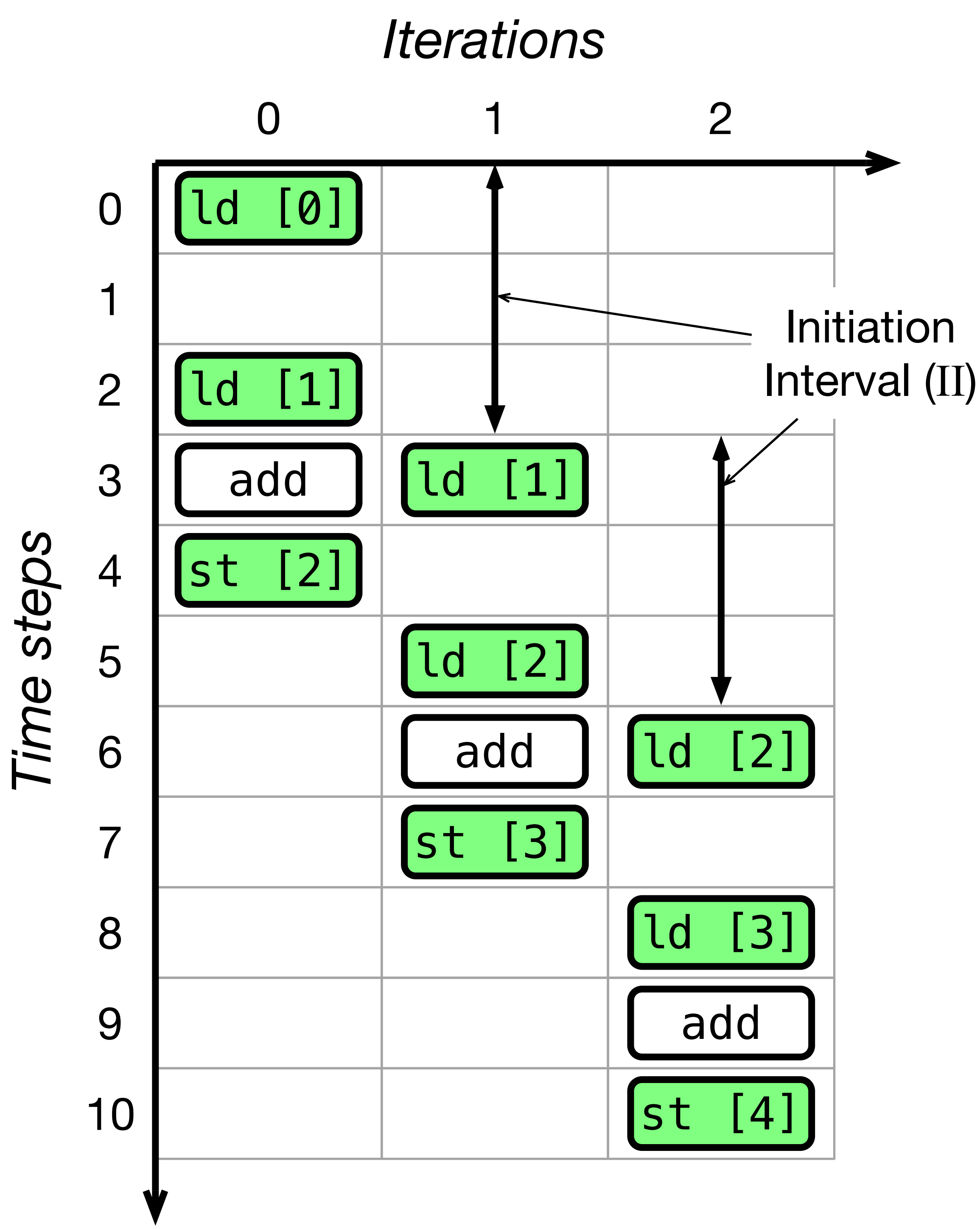

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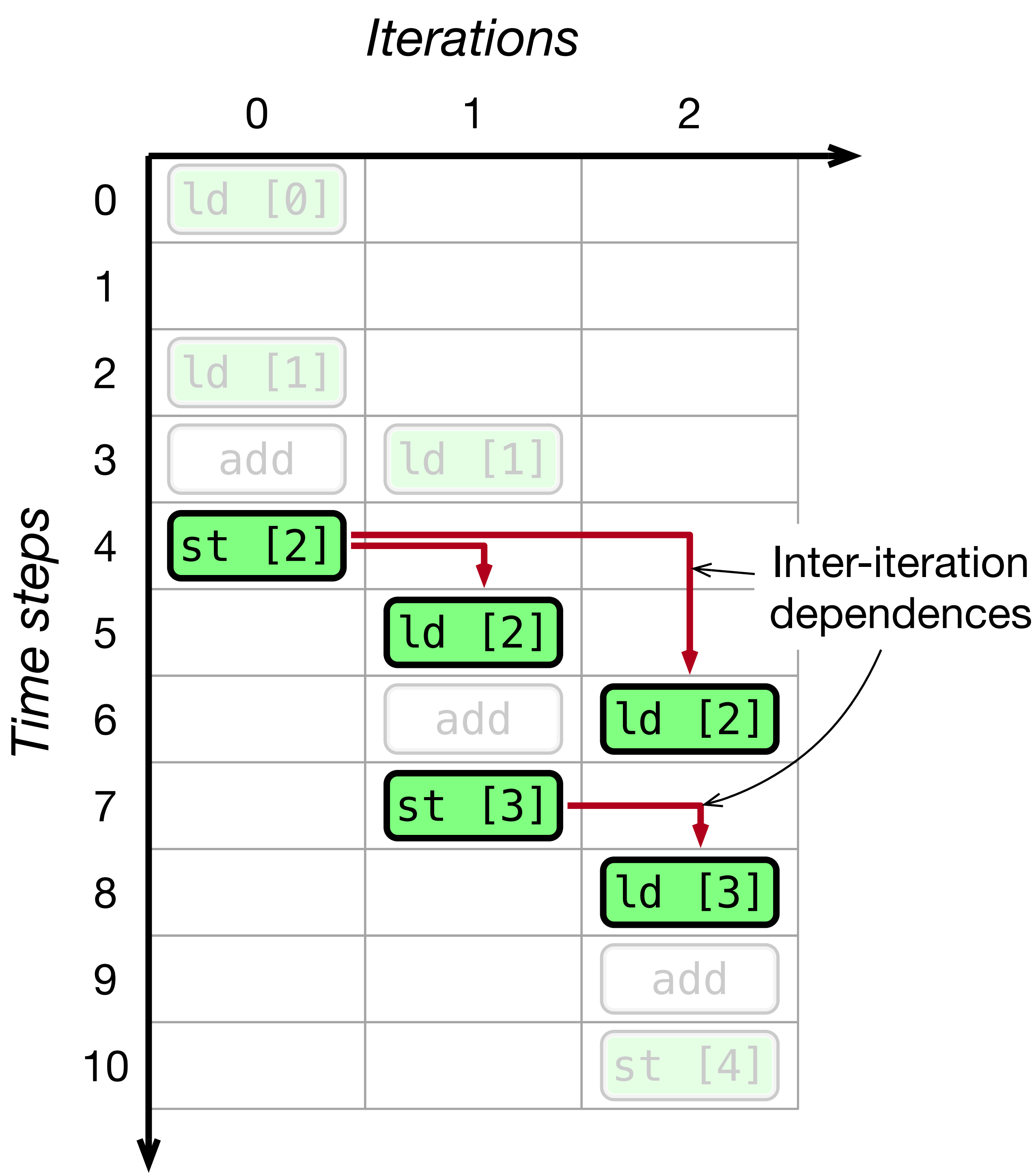

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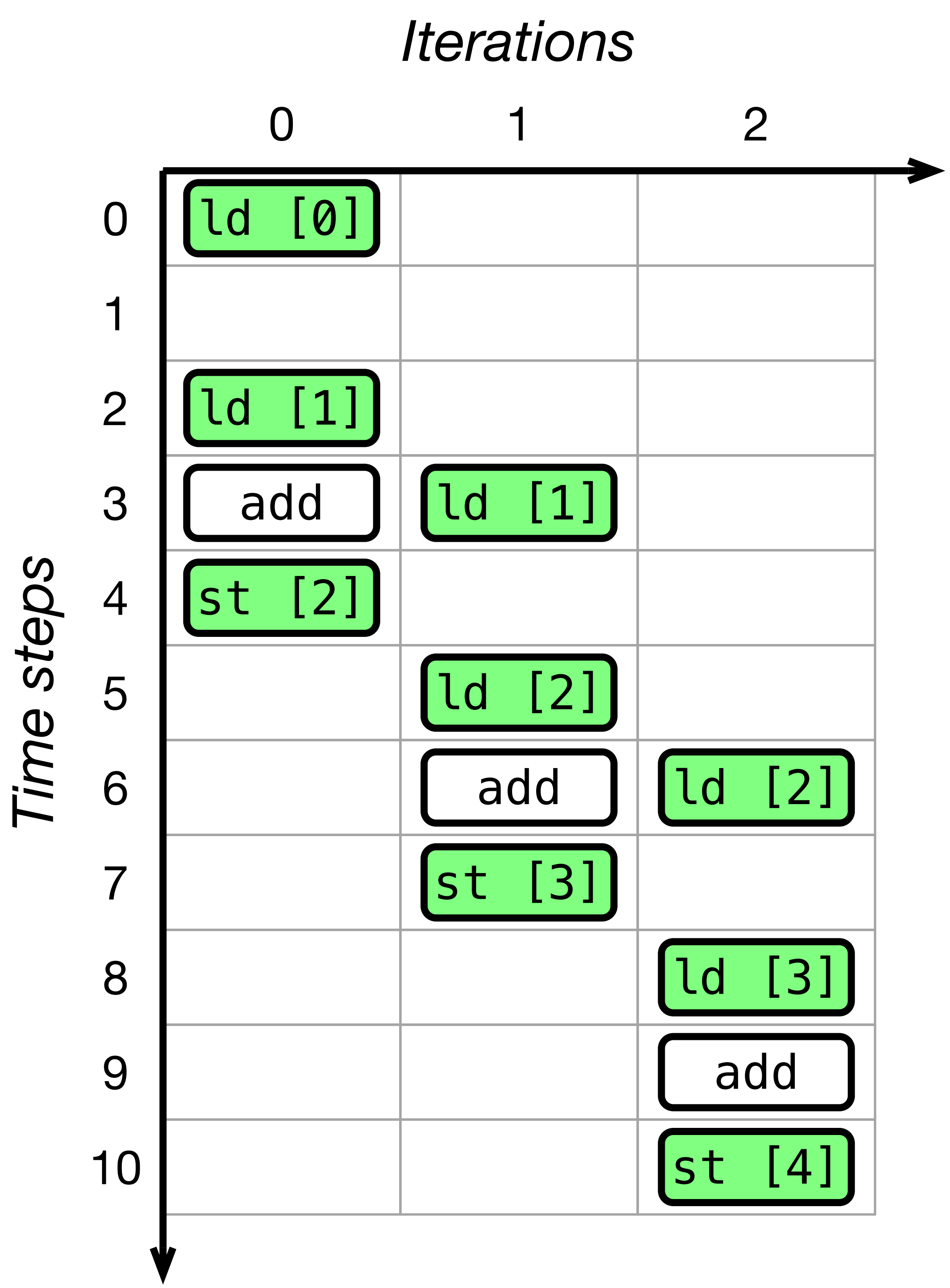

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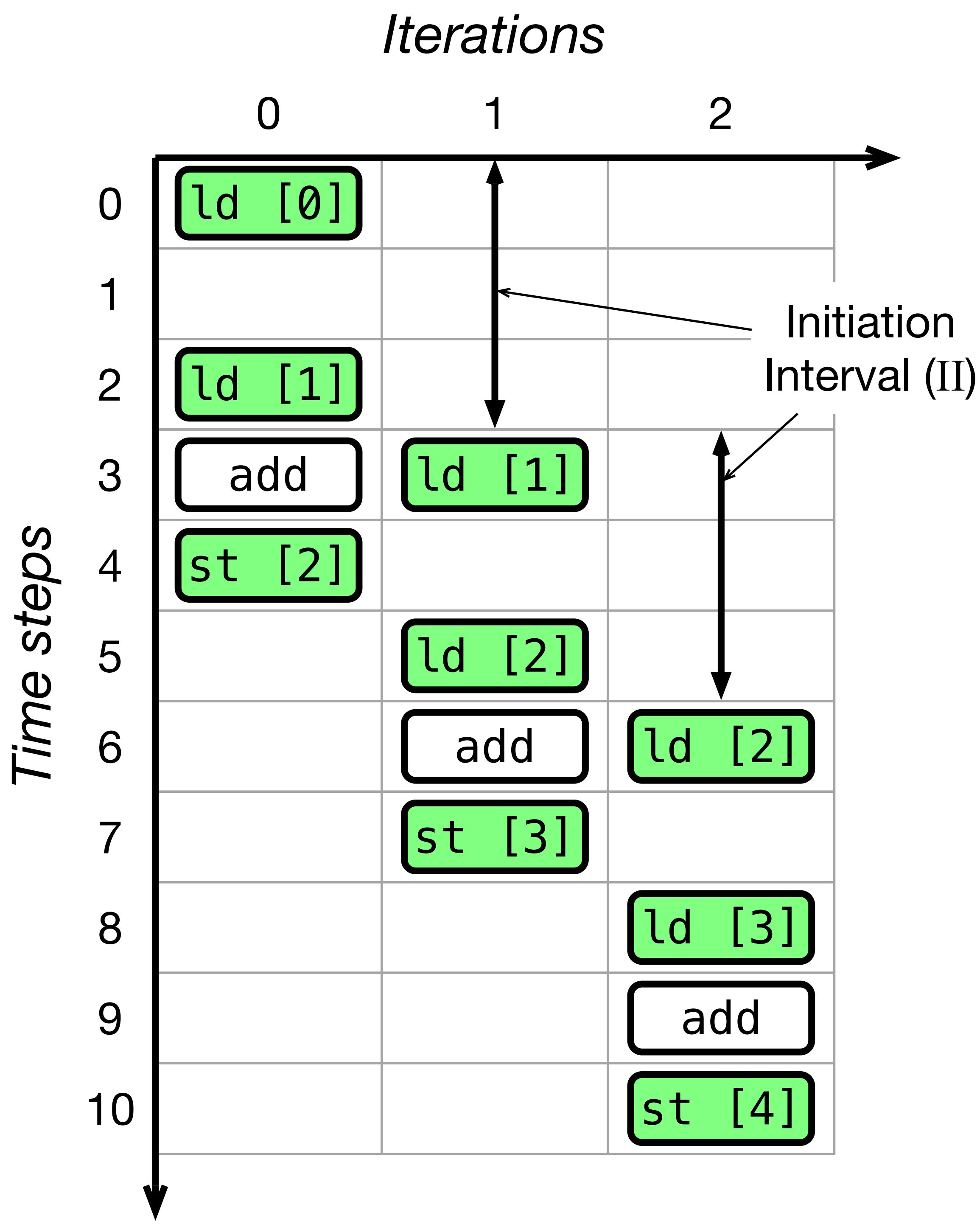

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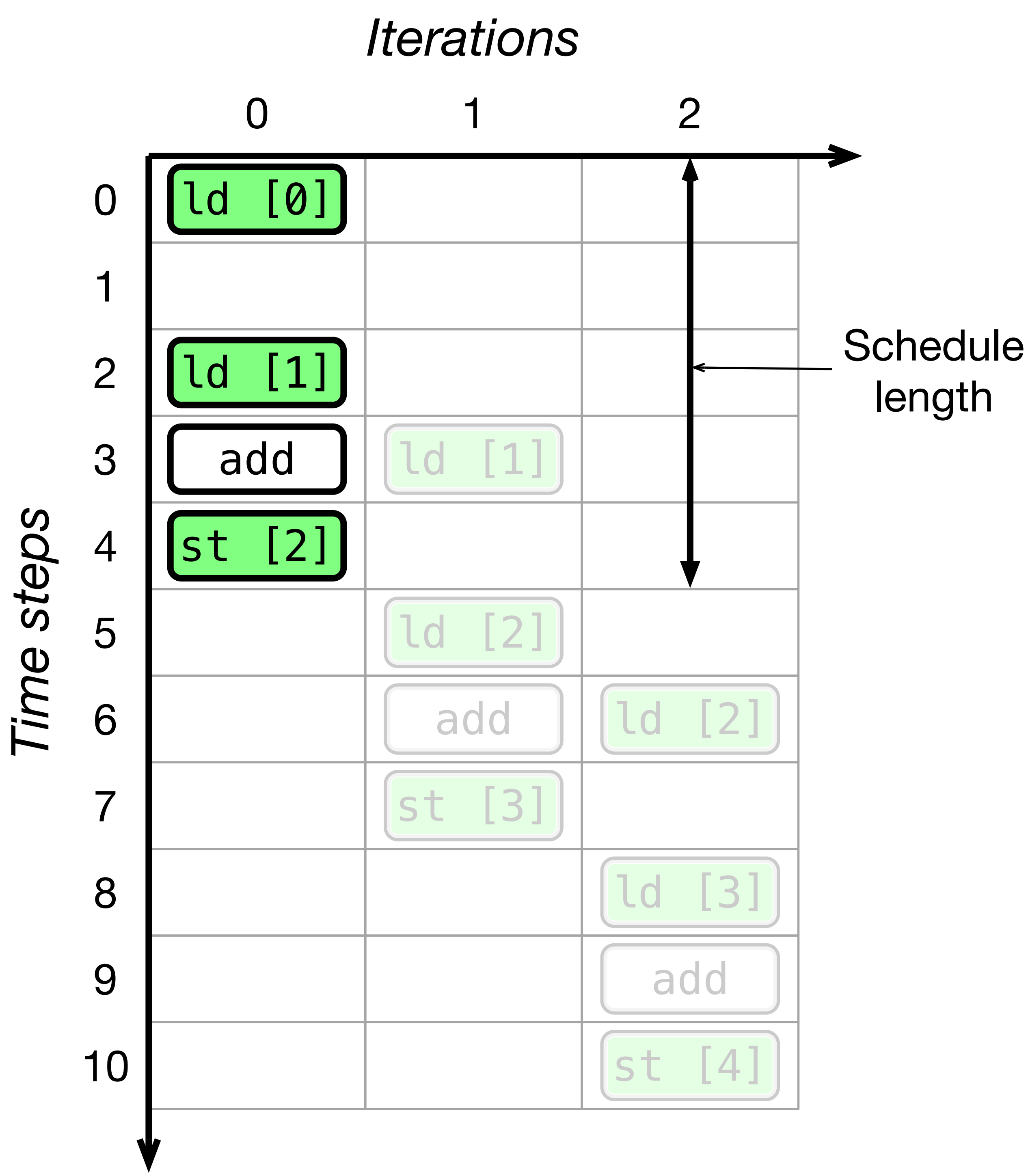

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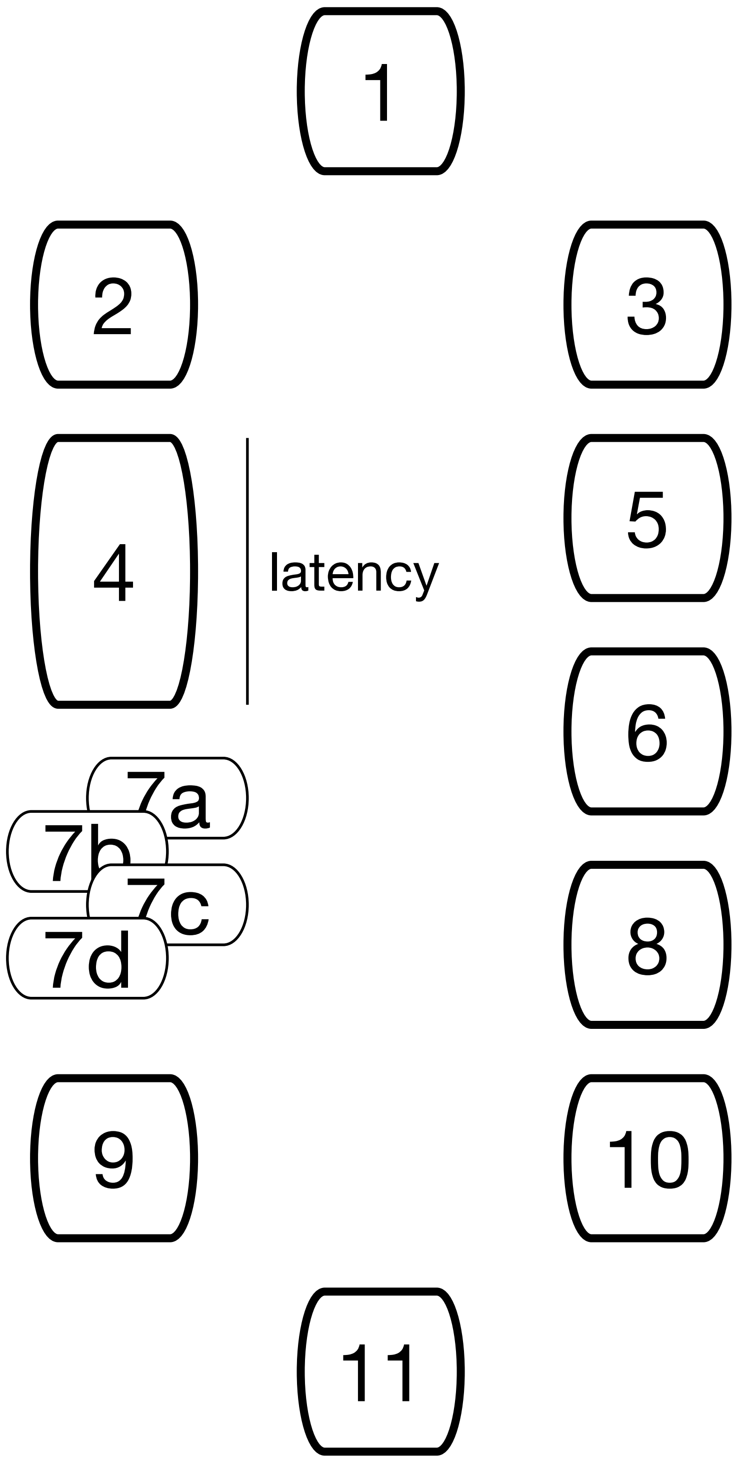

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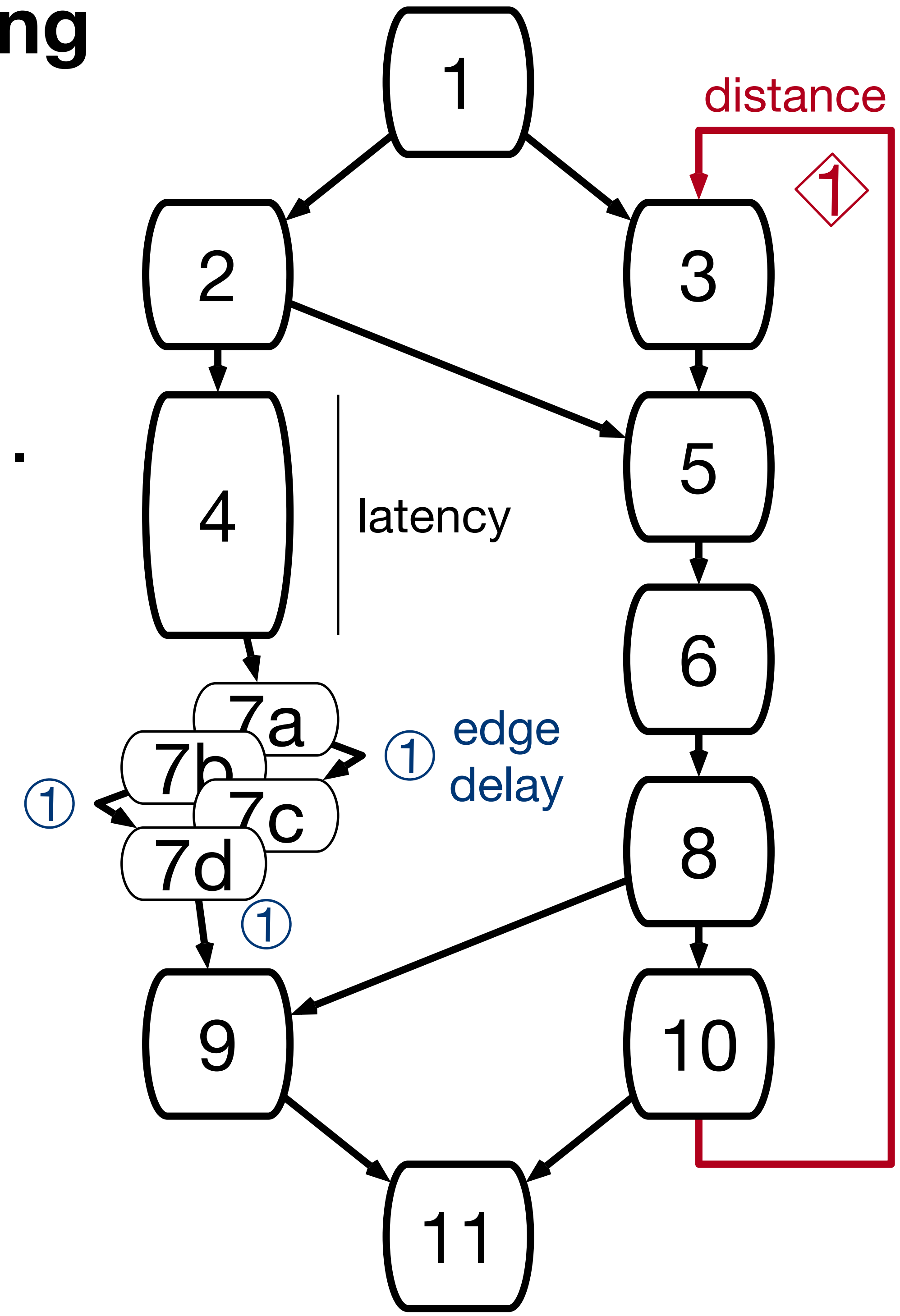

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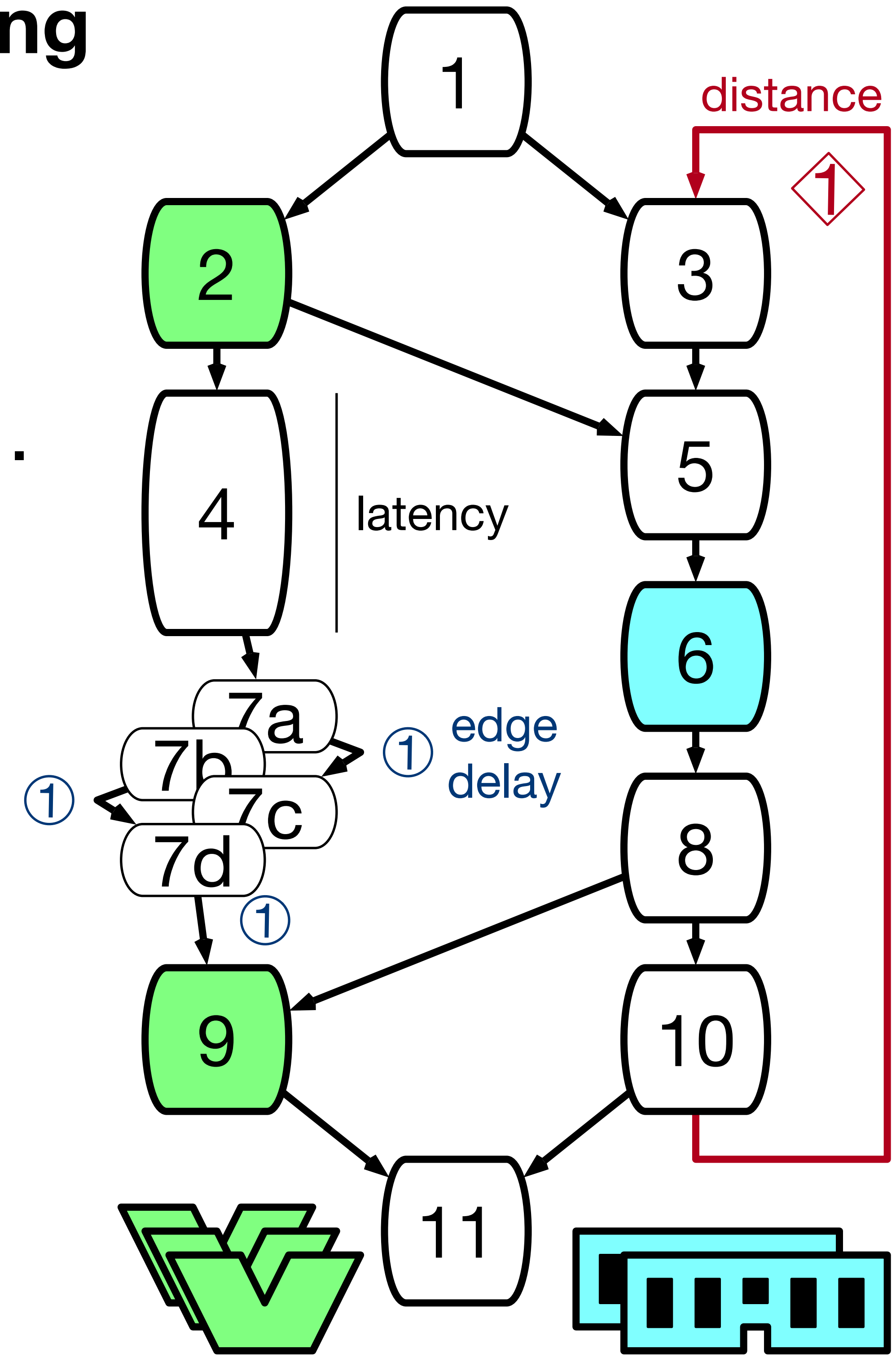
- Resource model
  - distinct types with given #units
  - unlimited operations



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  - High-level synthesis (**HLS**)
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    - **not all** operations are **resource-constrained**
- Observed scalability issues in highly-tuned approach

→ *Can't we just simplify the problem?*

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  - 1578 edges
    - 63% non-dataflow edges (memory dependences, chaining control)

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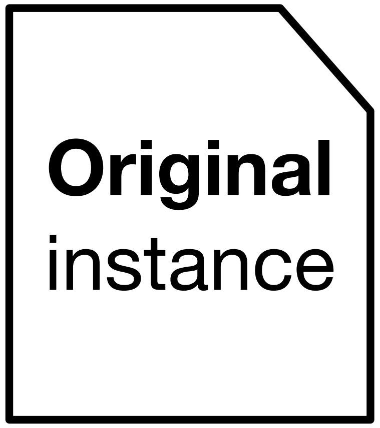
  - find subgraphs of **non-critical** operations
  - replace by a single edge with appropriate delay

Approach Overview



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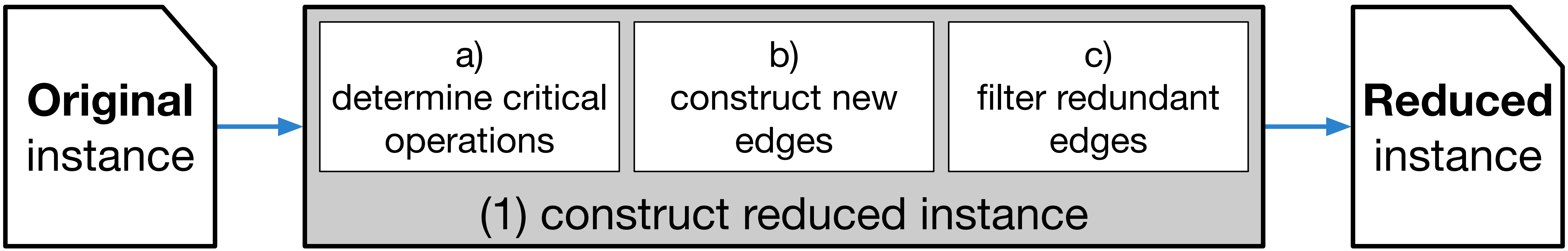

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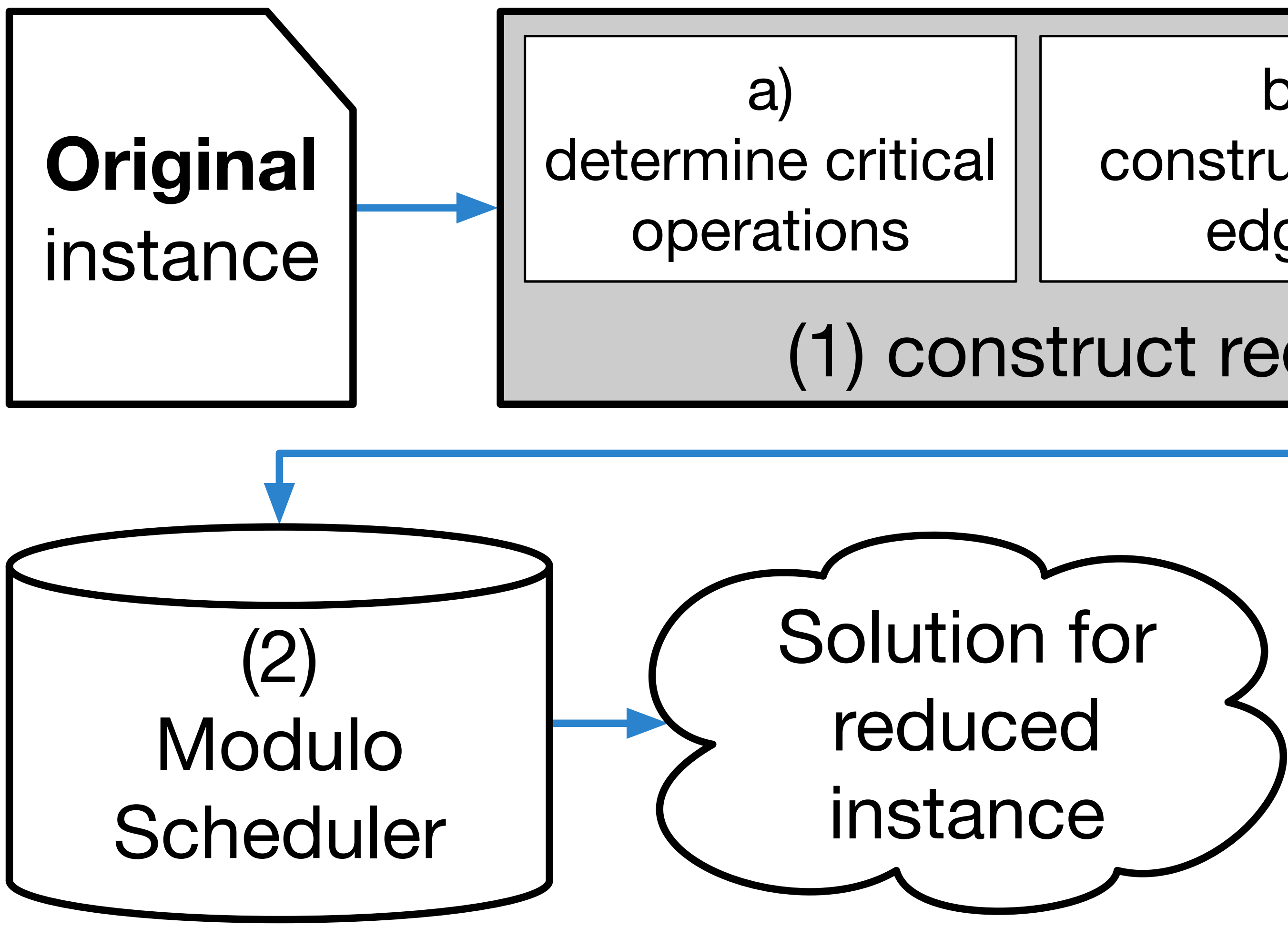

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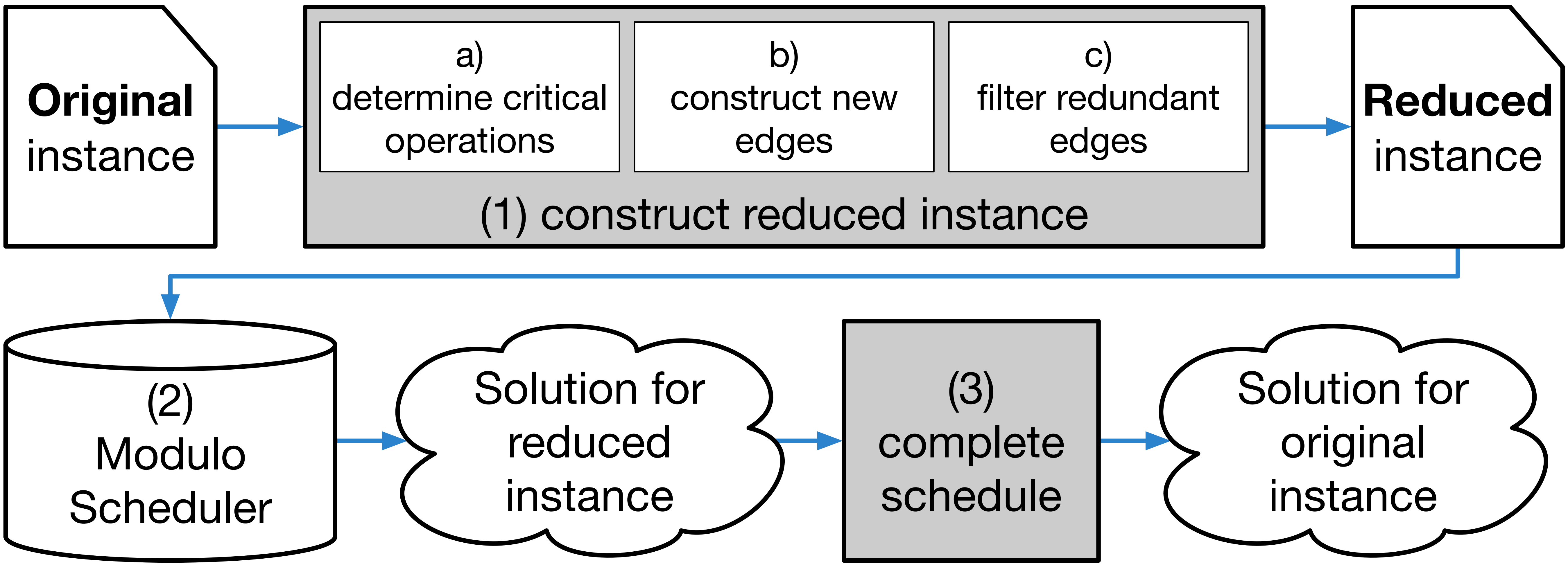
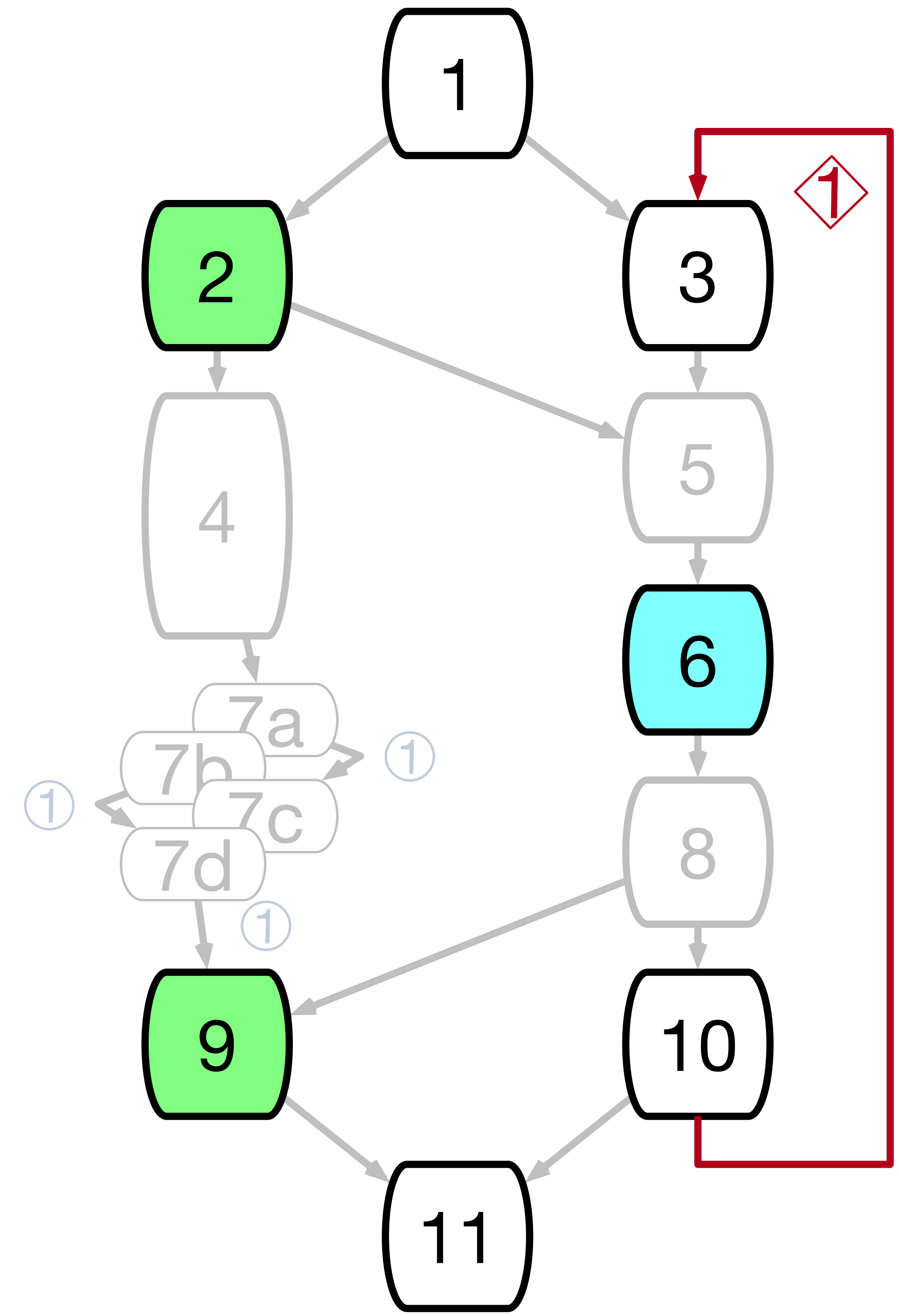


1a) Determine Critical Operations

- Feasibility



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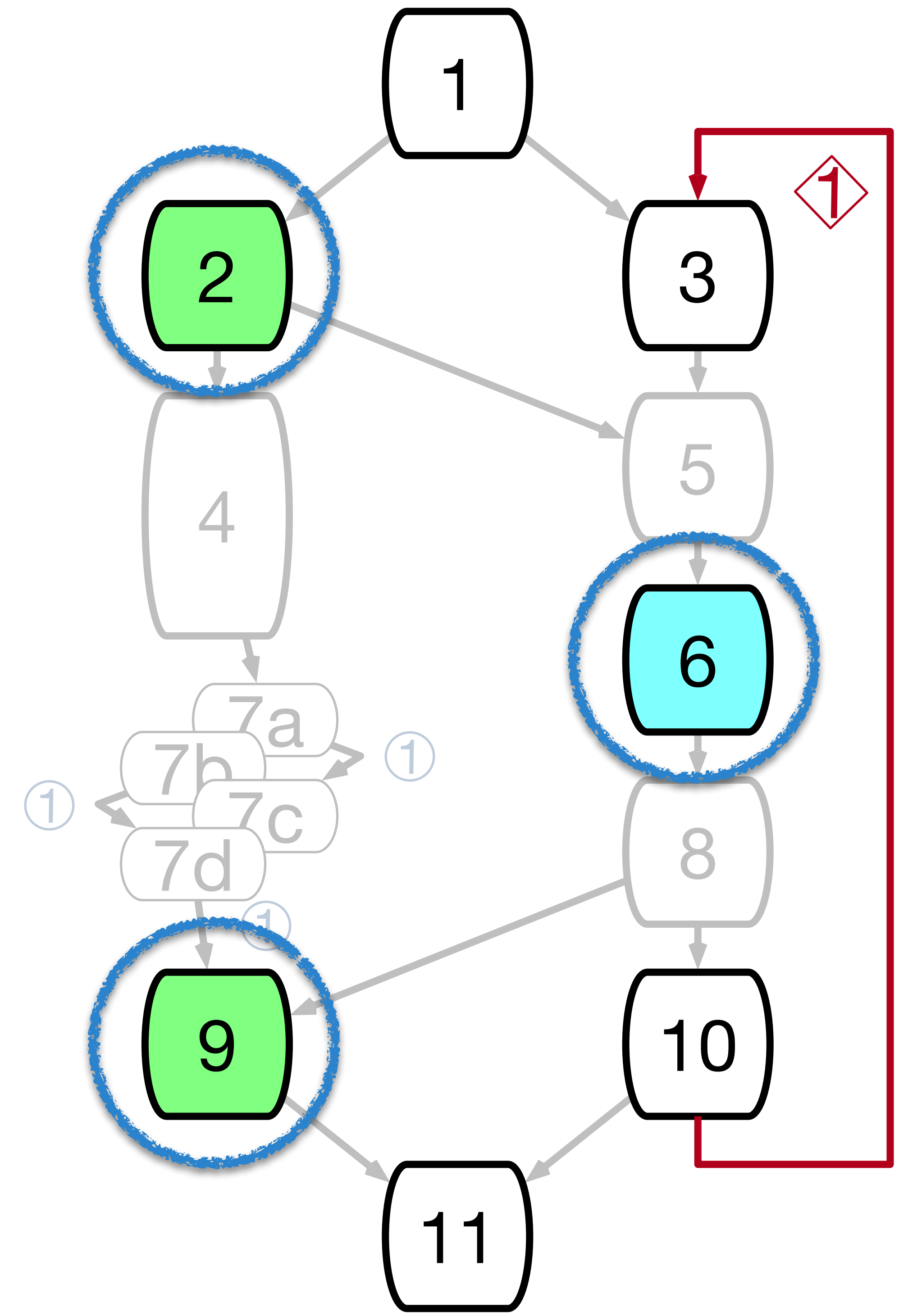

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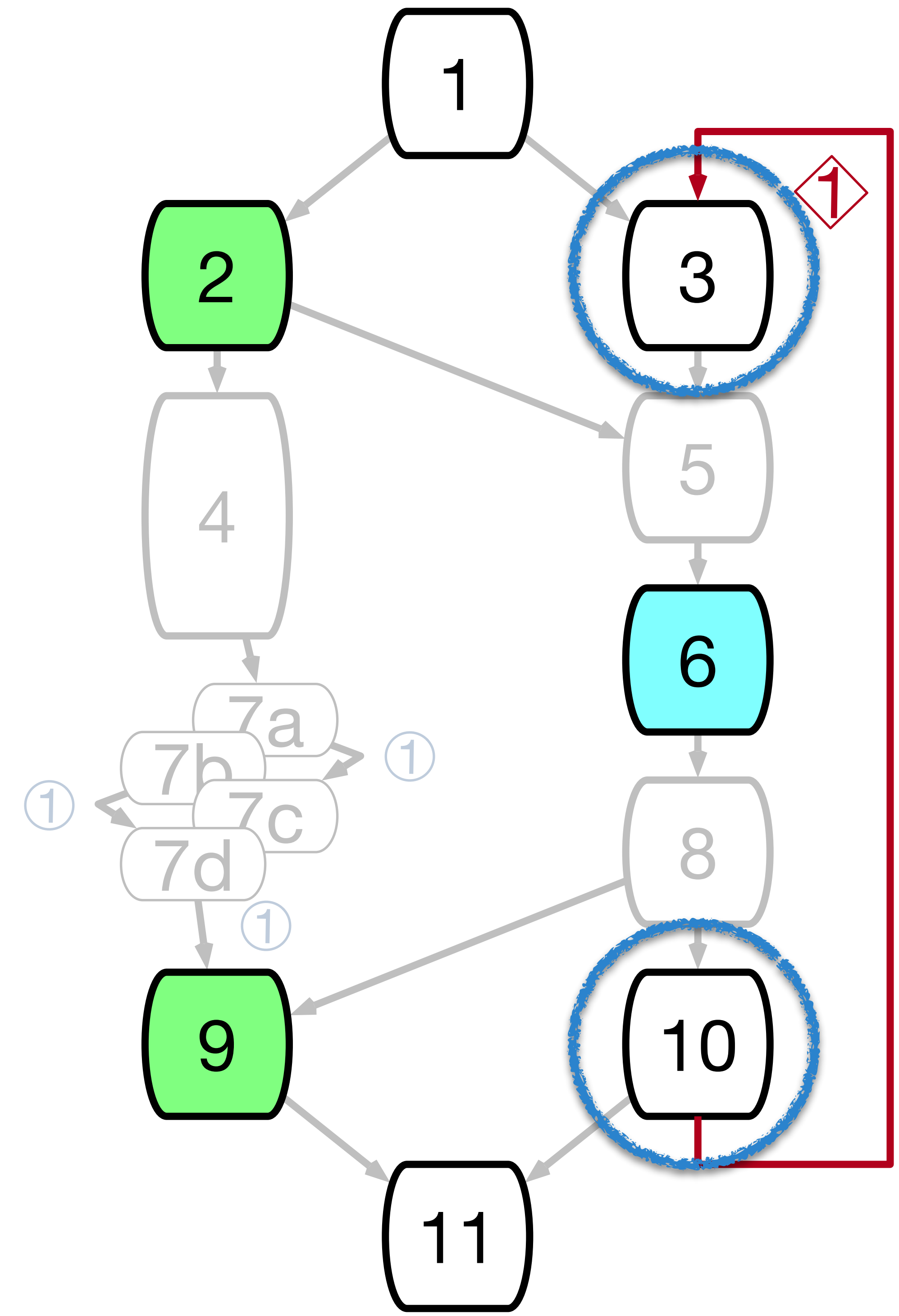

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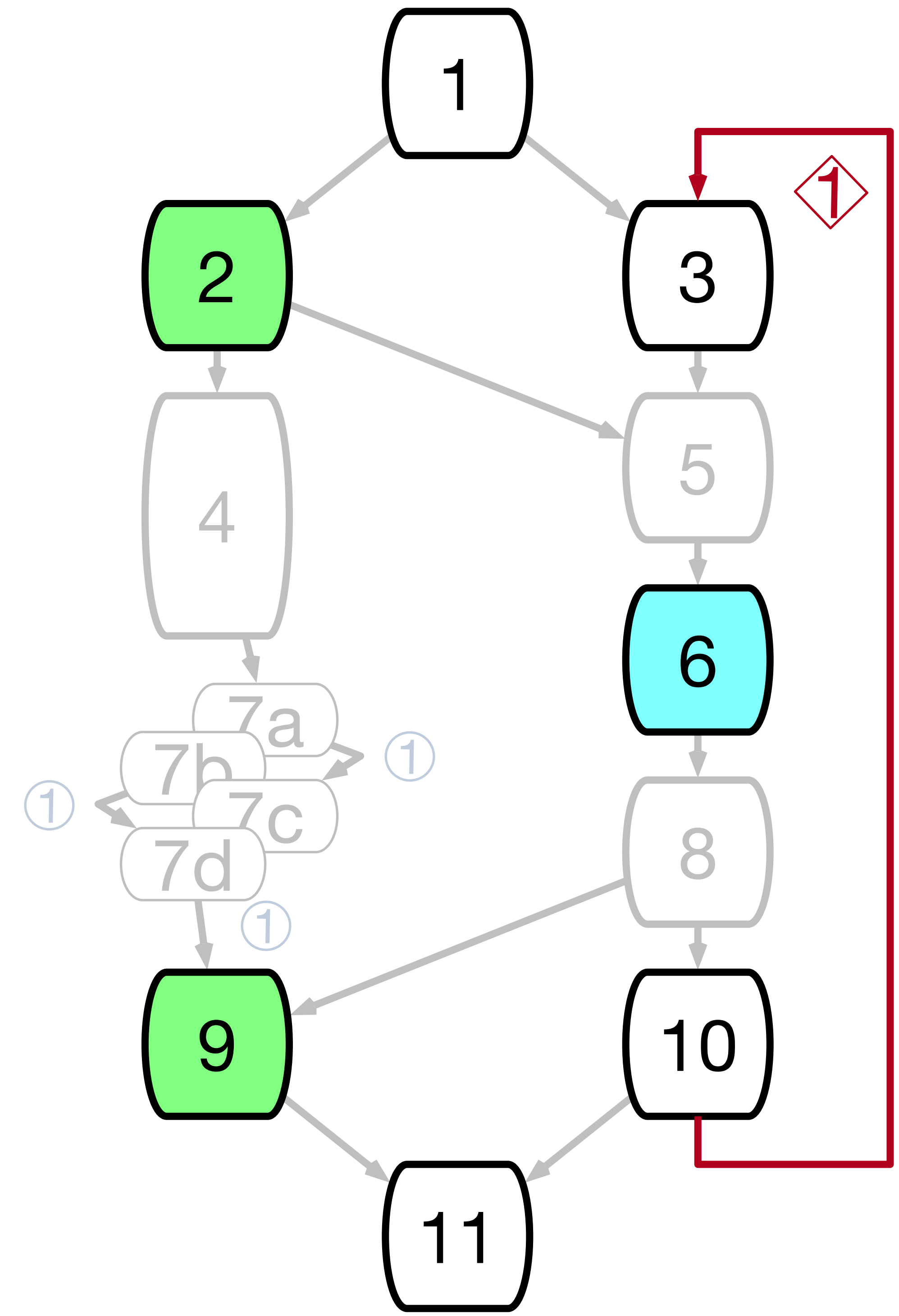

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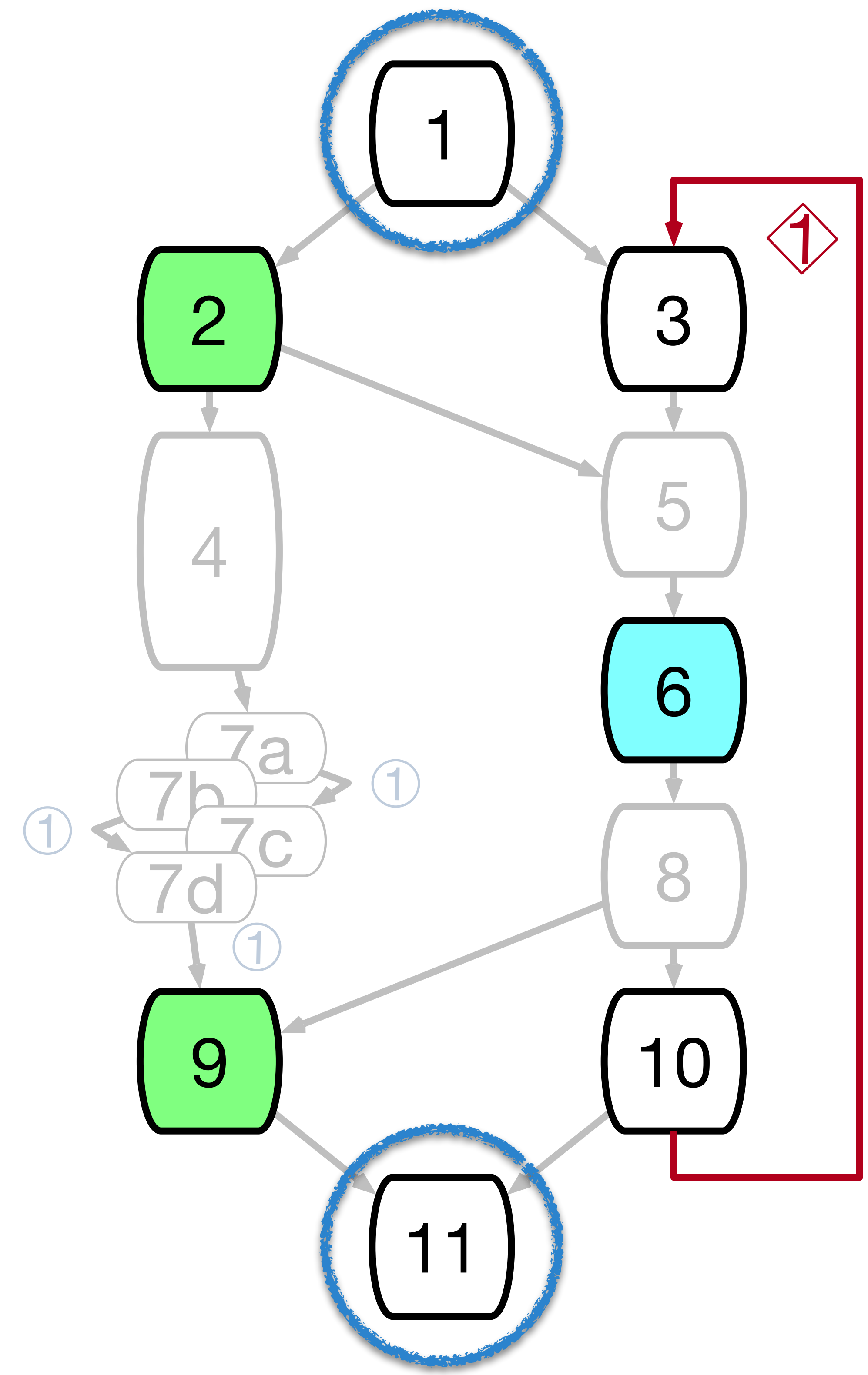
- Objective
  - source and sink nodes



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  - paths are **reset** when encountering a critical operation

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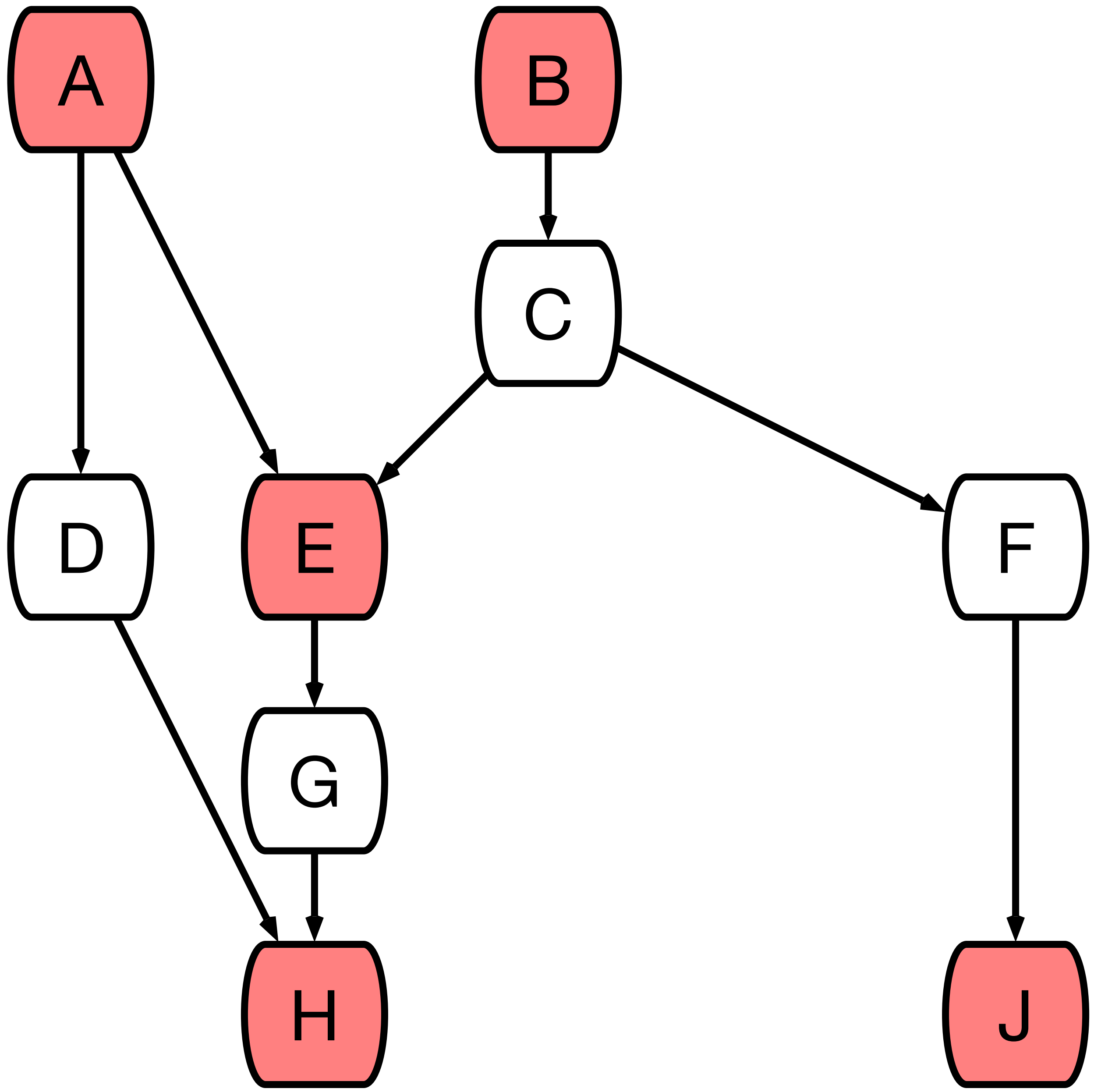
- Example of data-flow analysis



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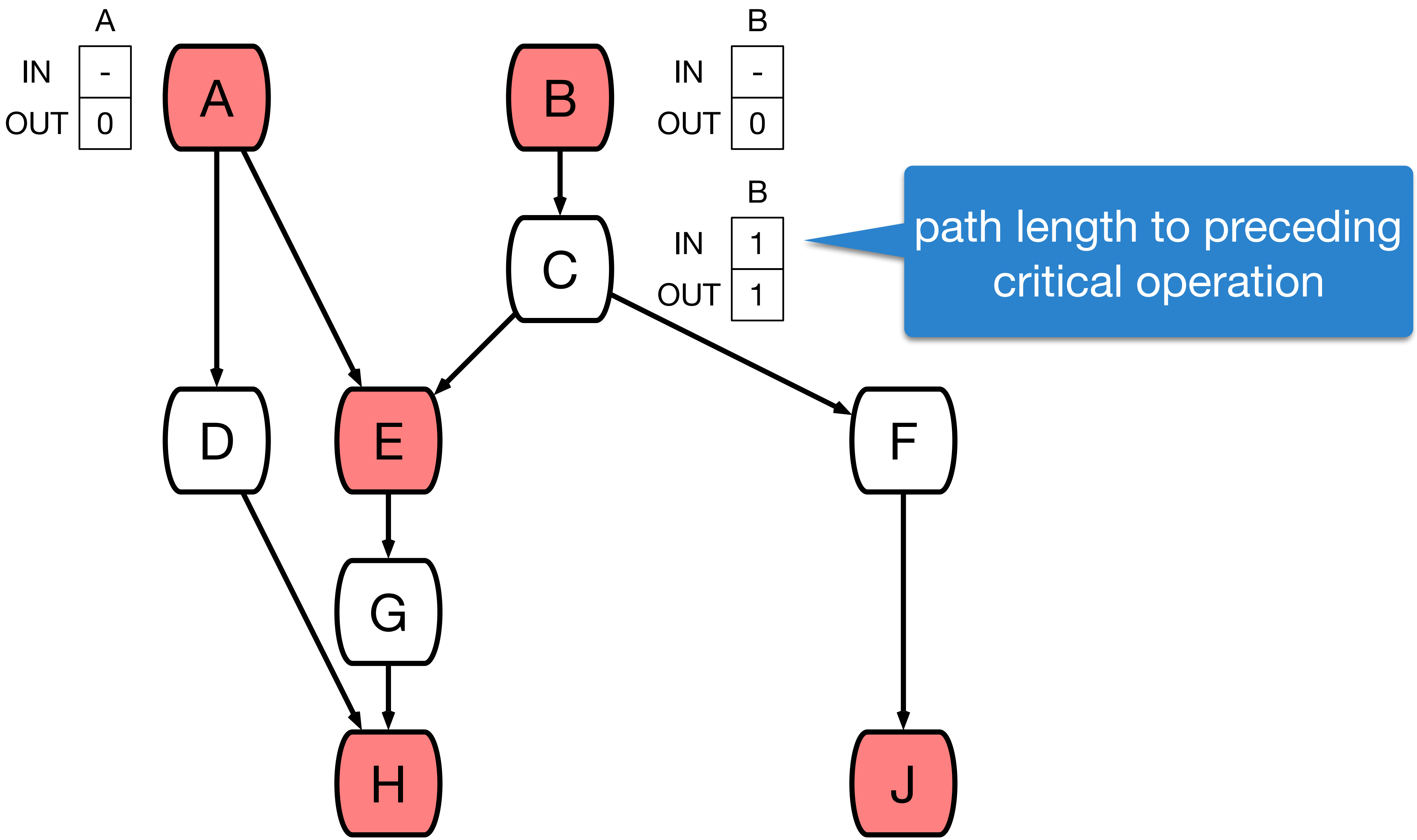

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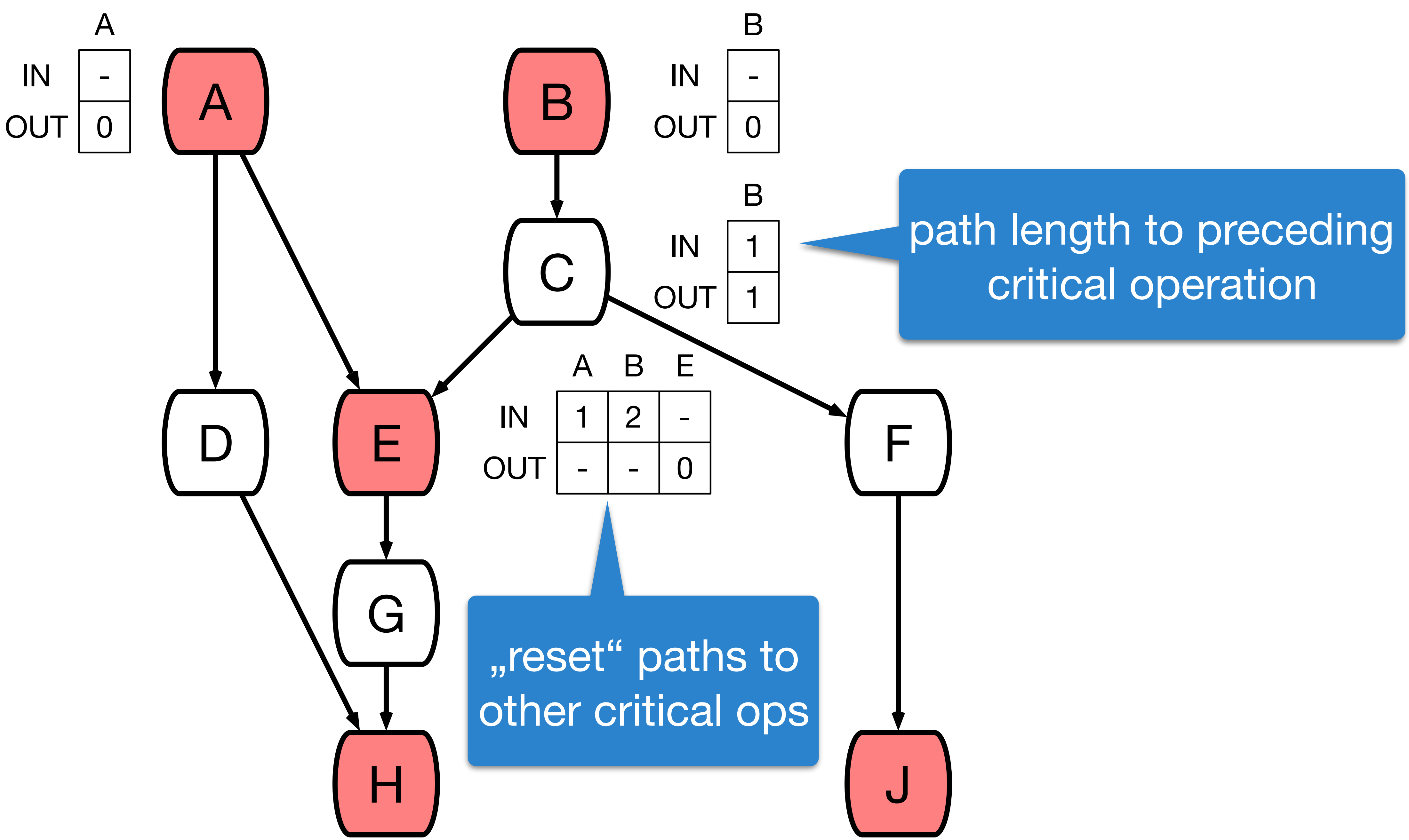

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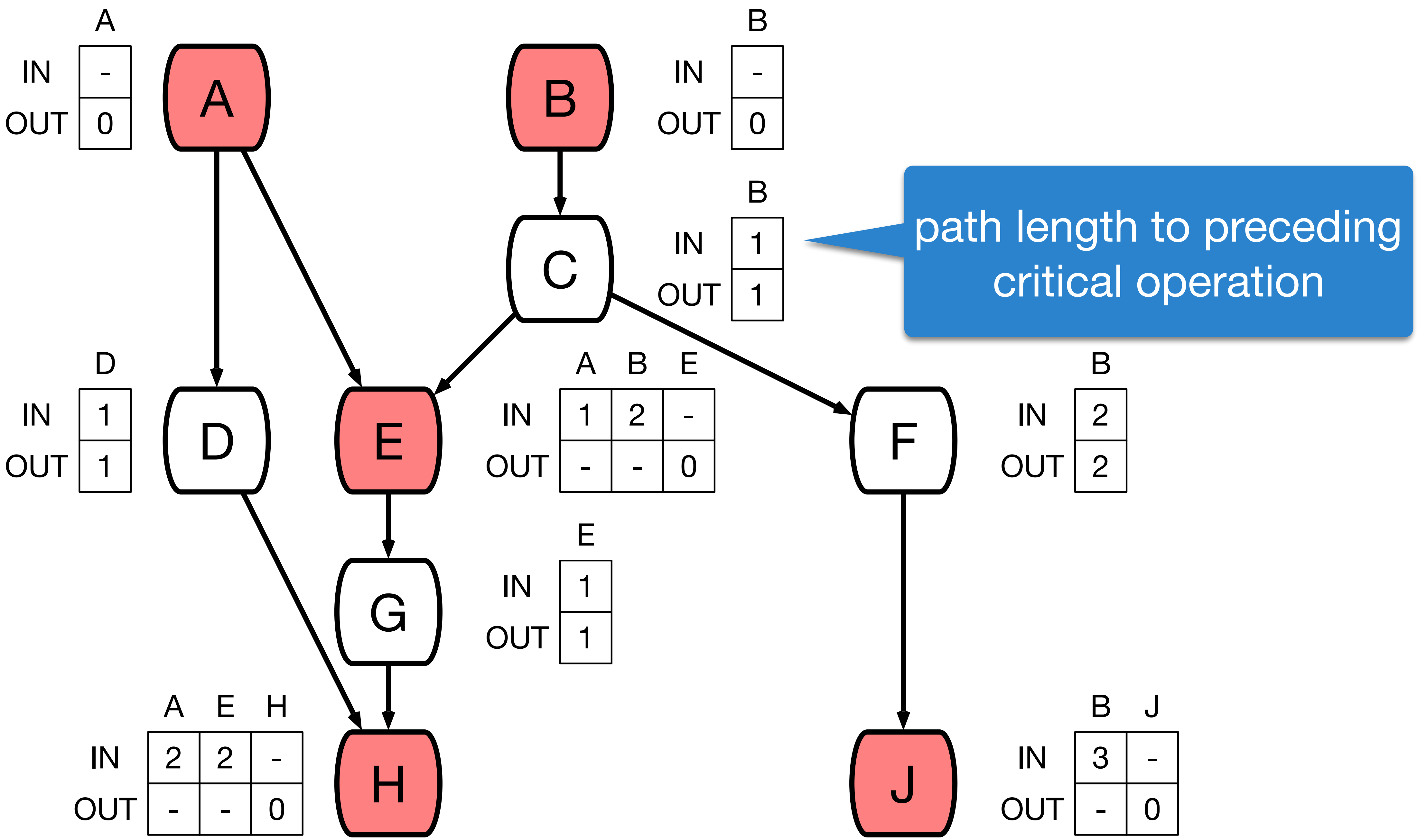

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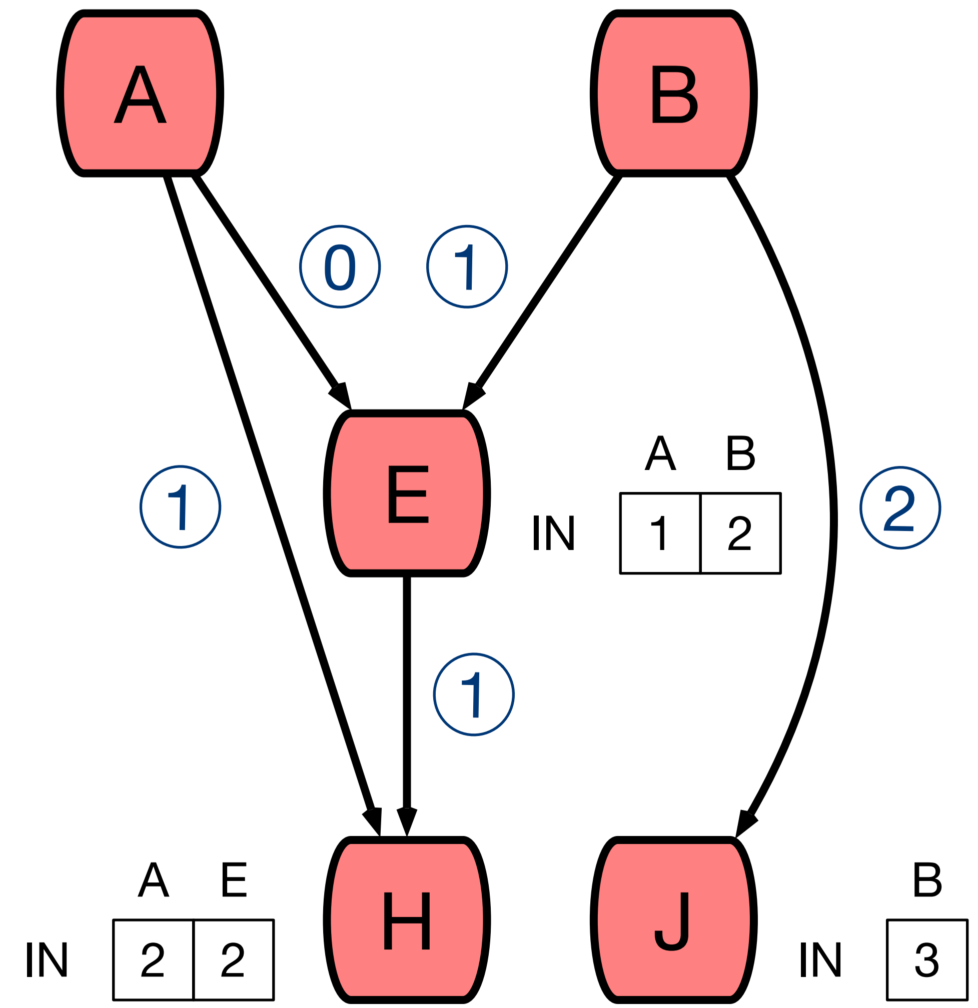

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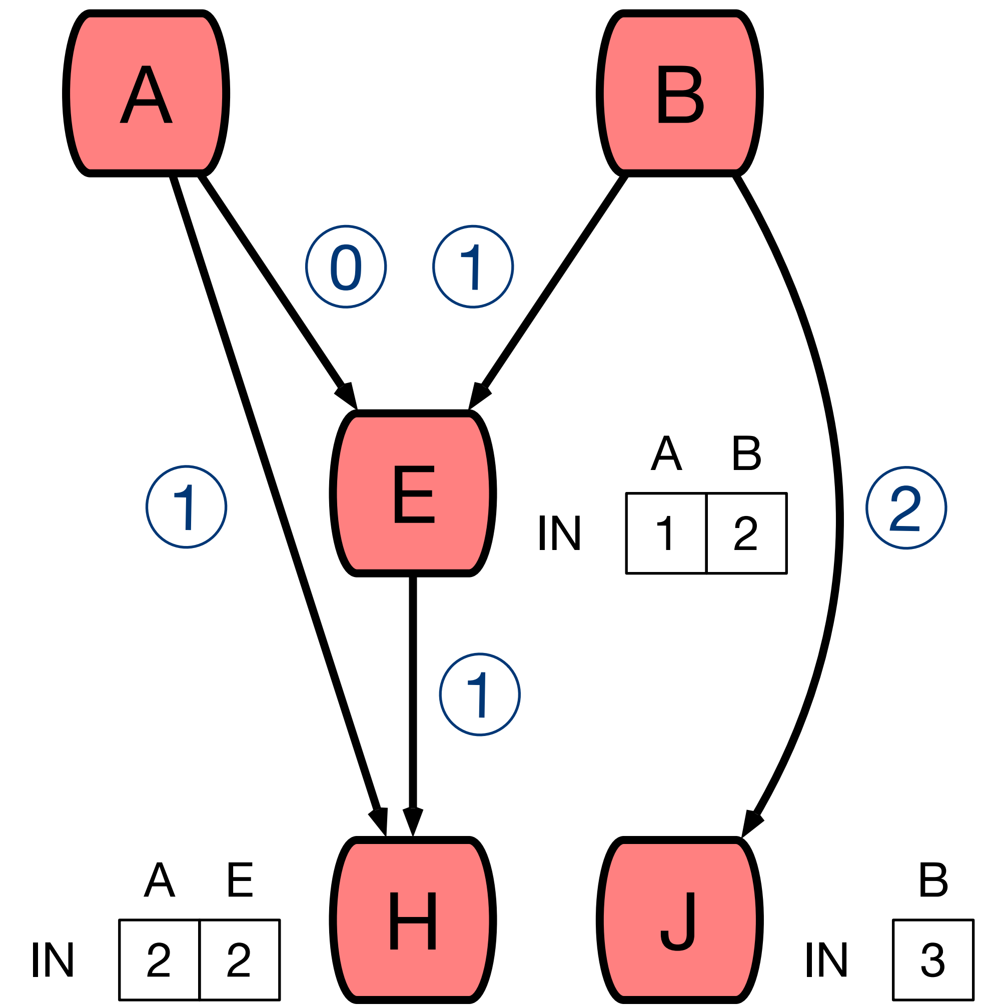

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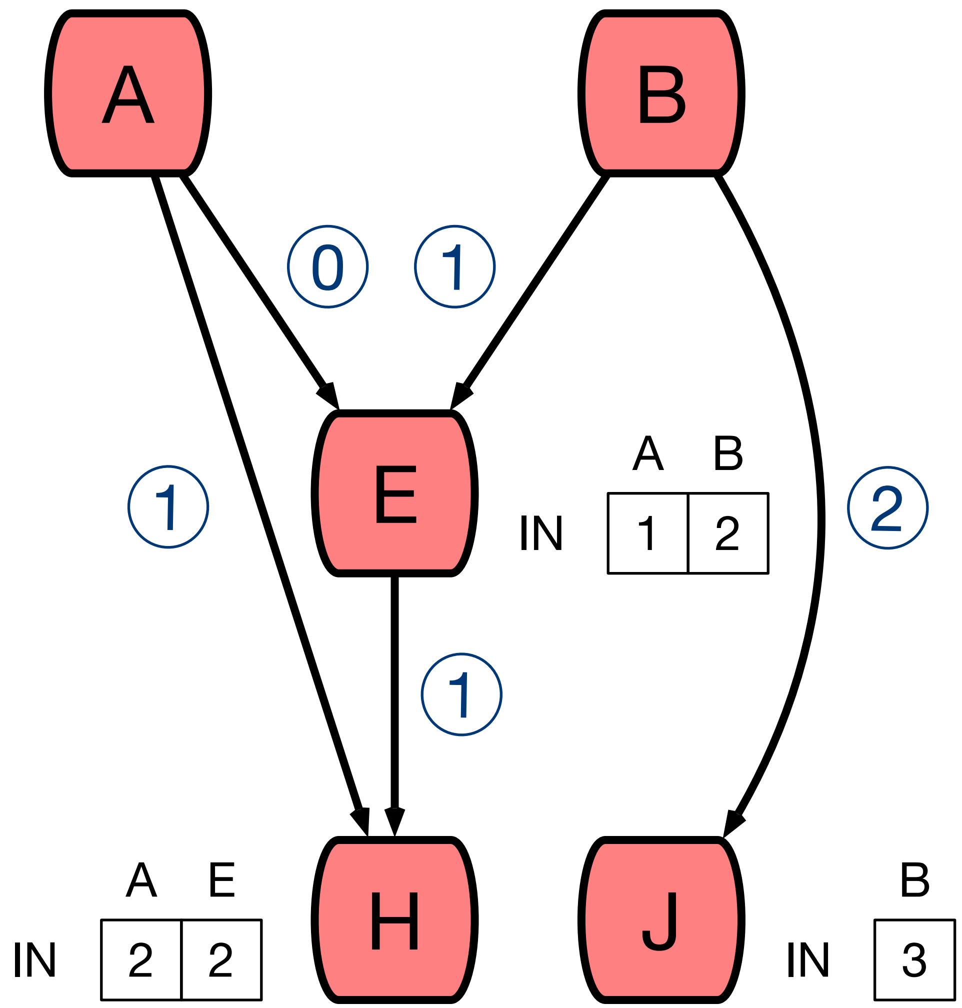
- Backedges are copied over from original instance



1c) Filter Redundant Edges

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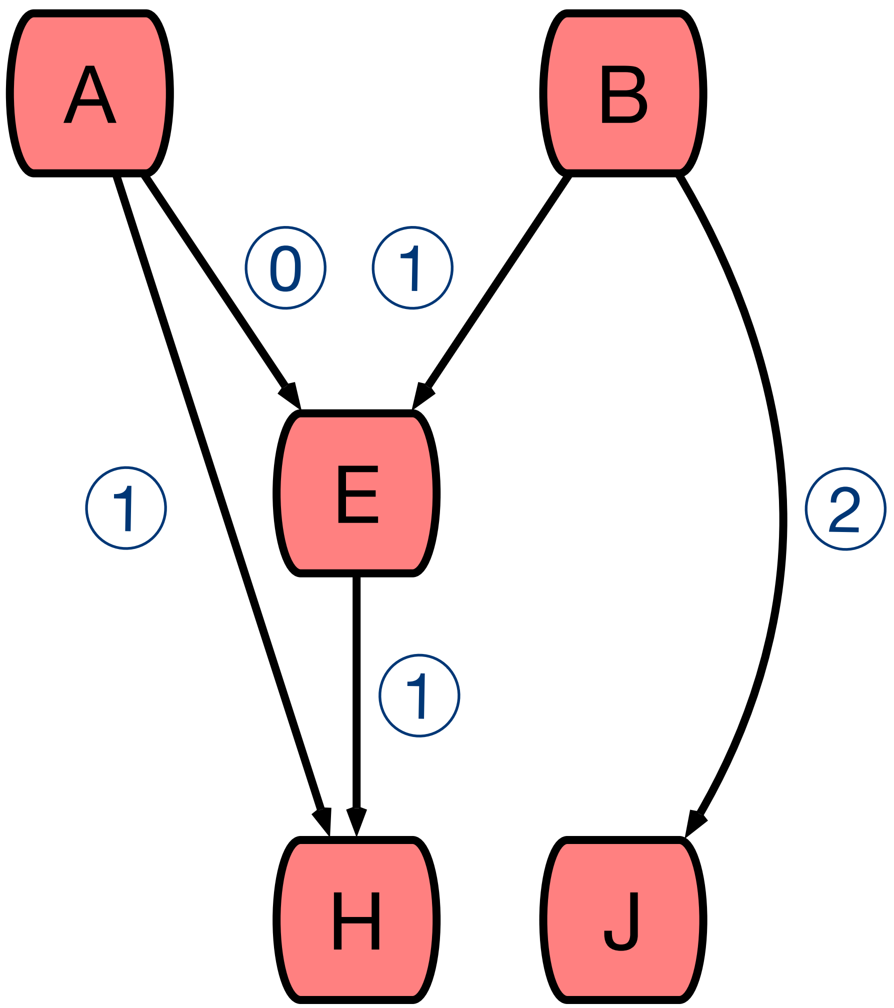

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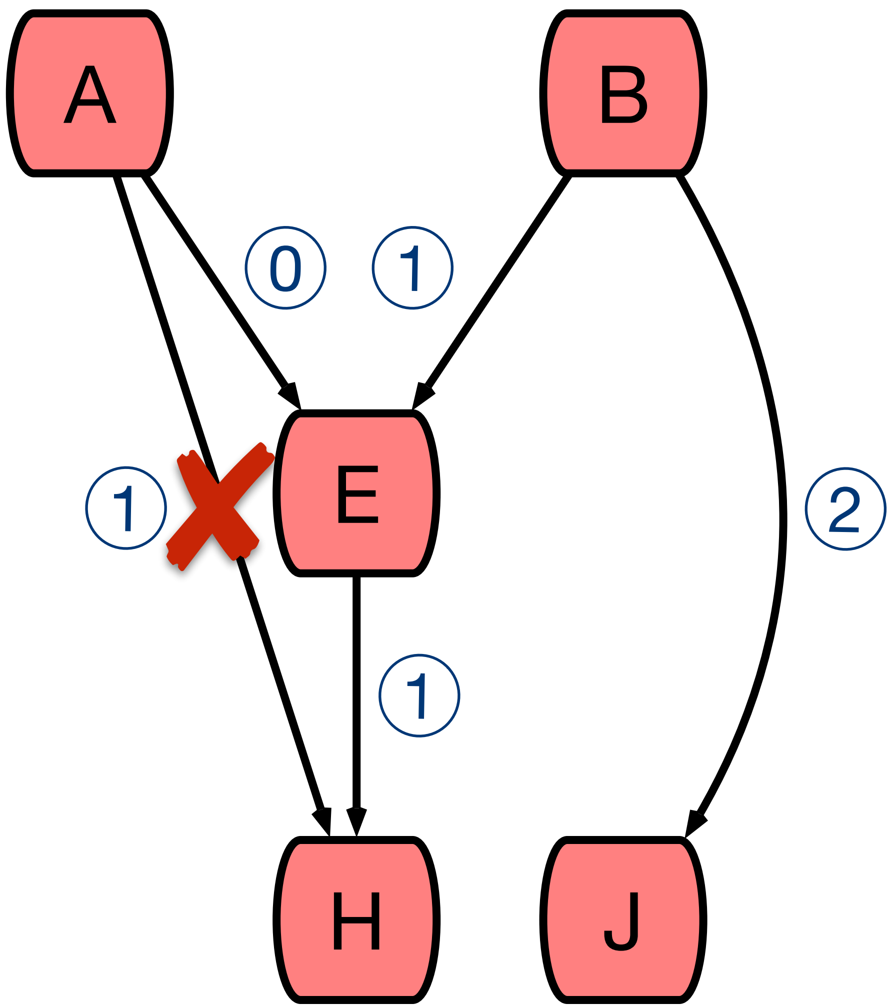

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→ different approaches to model operations’ start times and resource limits

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  → easy (polynomially) to solve, because no longer resource-constrained

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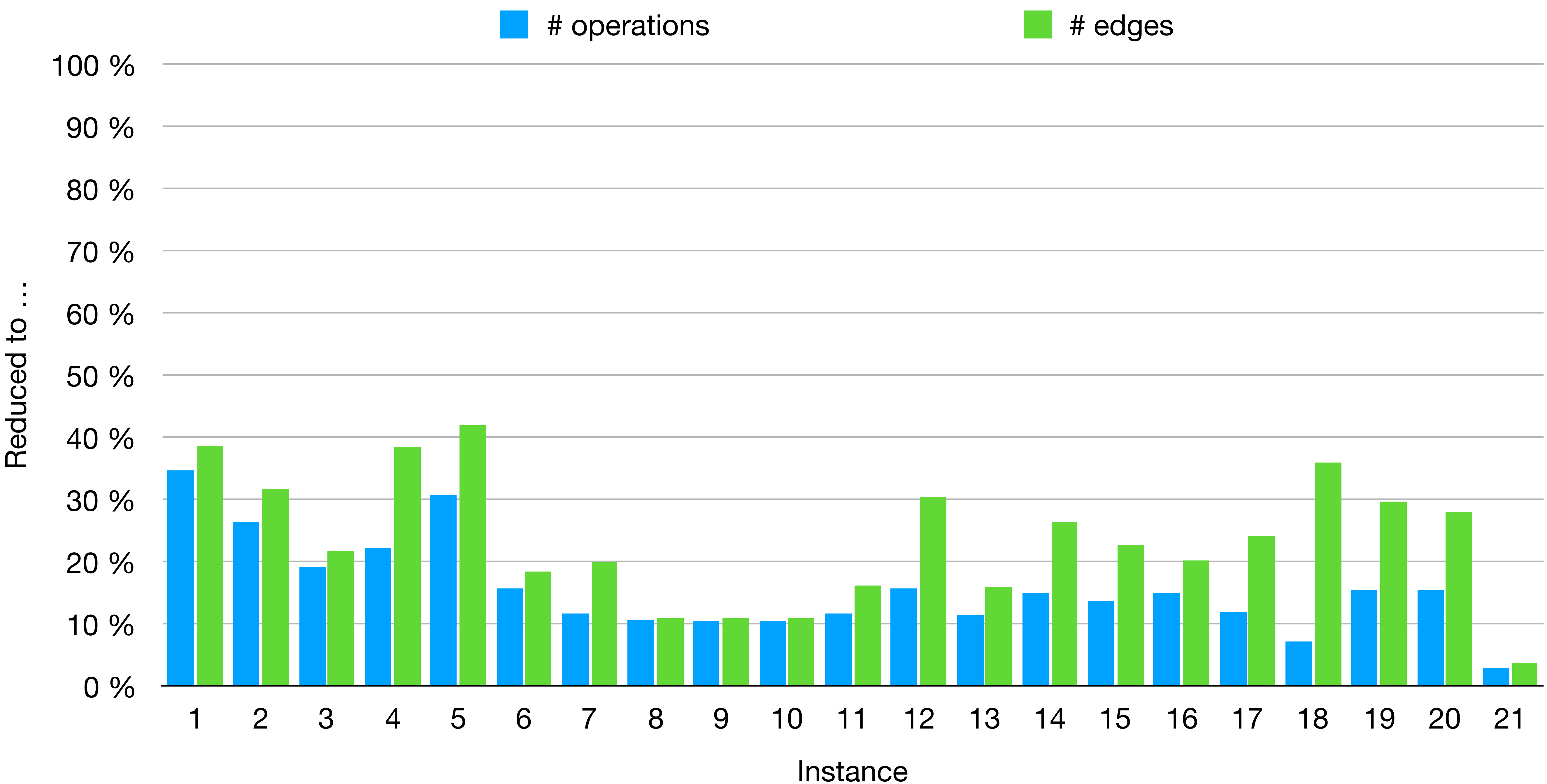
- Ran on 24-core Xeon E5-2680 v3, 2.8 GHz, 64 GB RAM

Results: Graph Reduction



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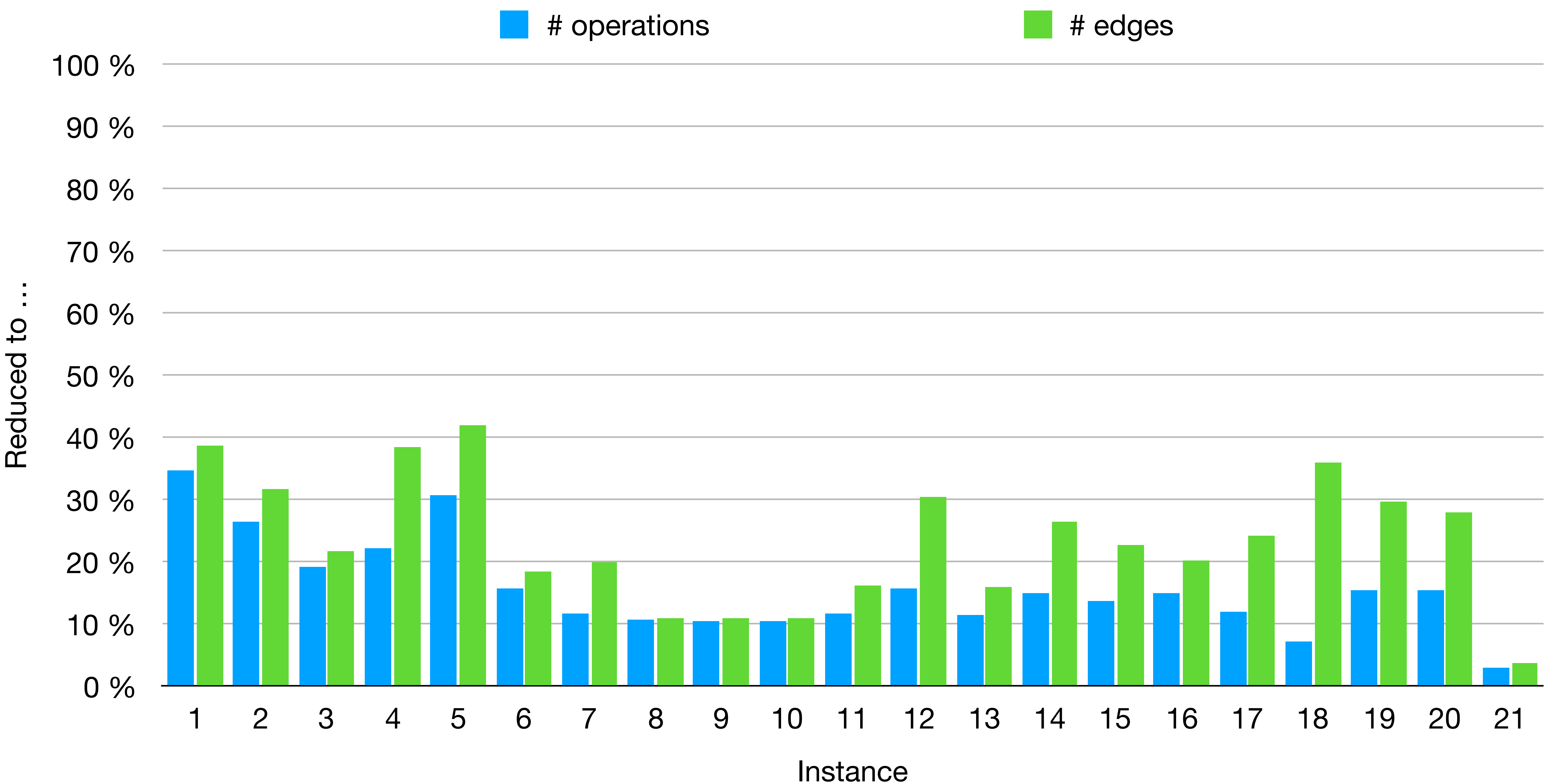
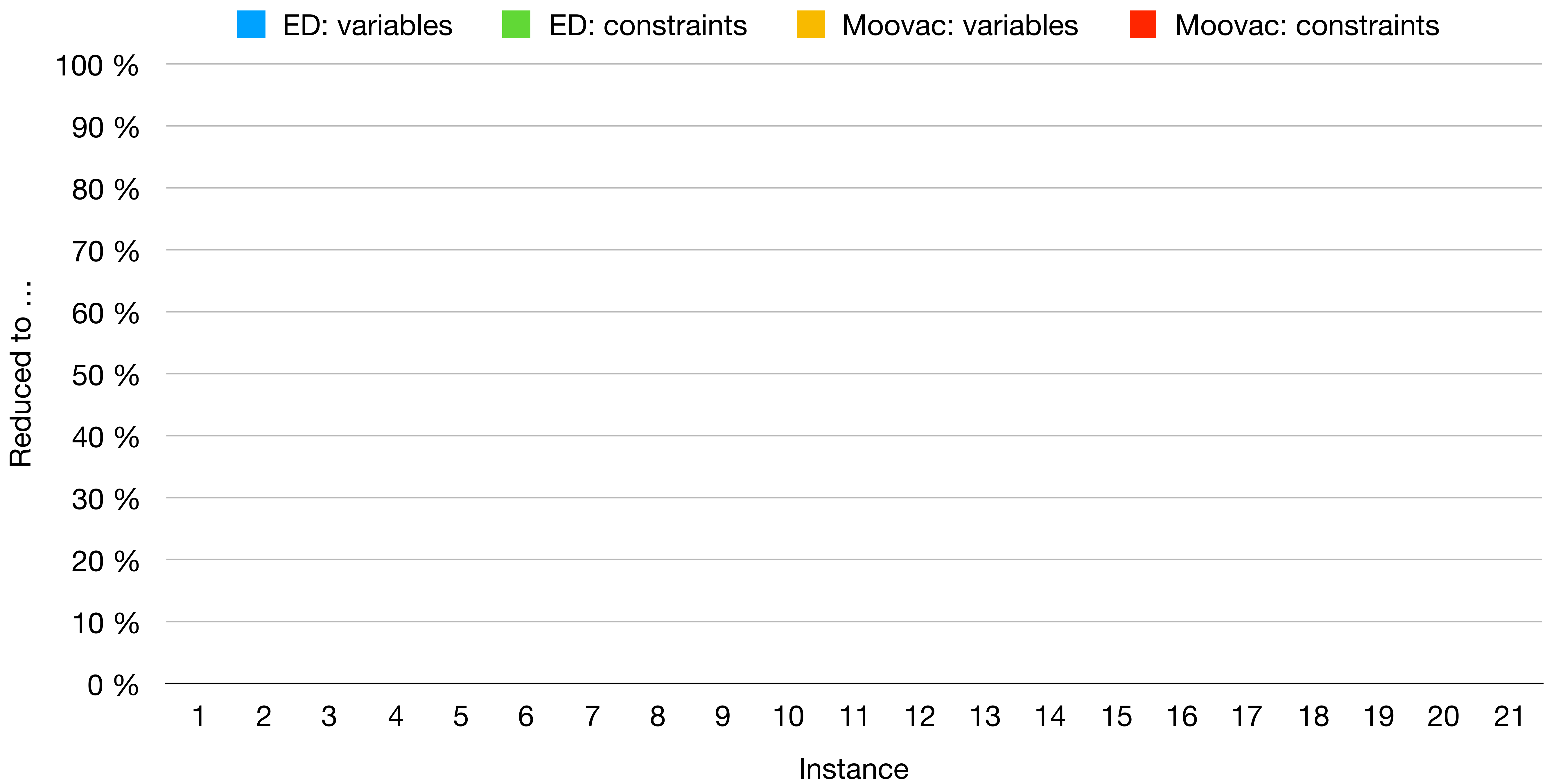


- Time for complexity reduction: **always < 0.5 sec**

Results: ILP Reduction



Mean #
Variables
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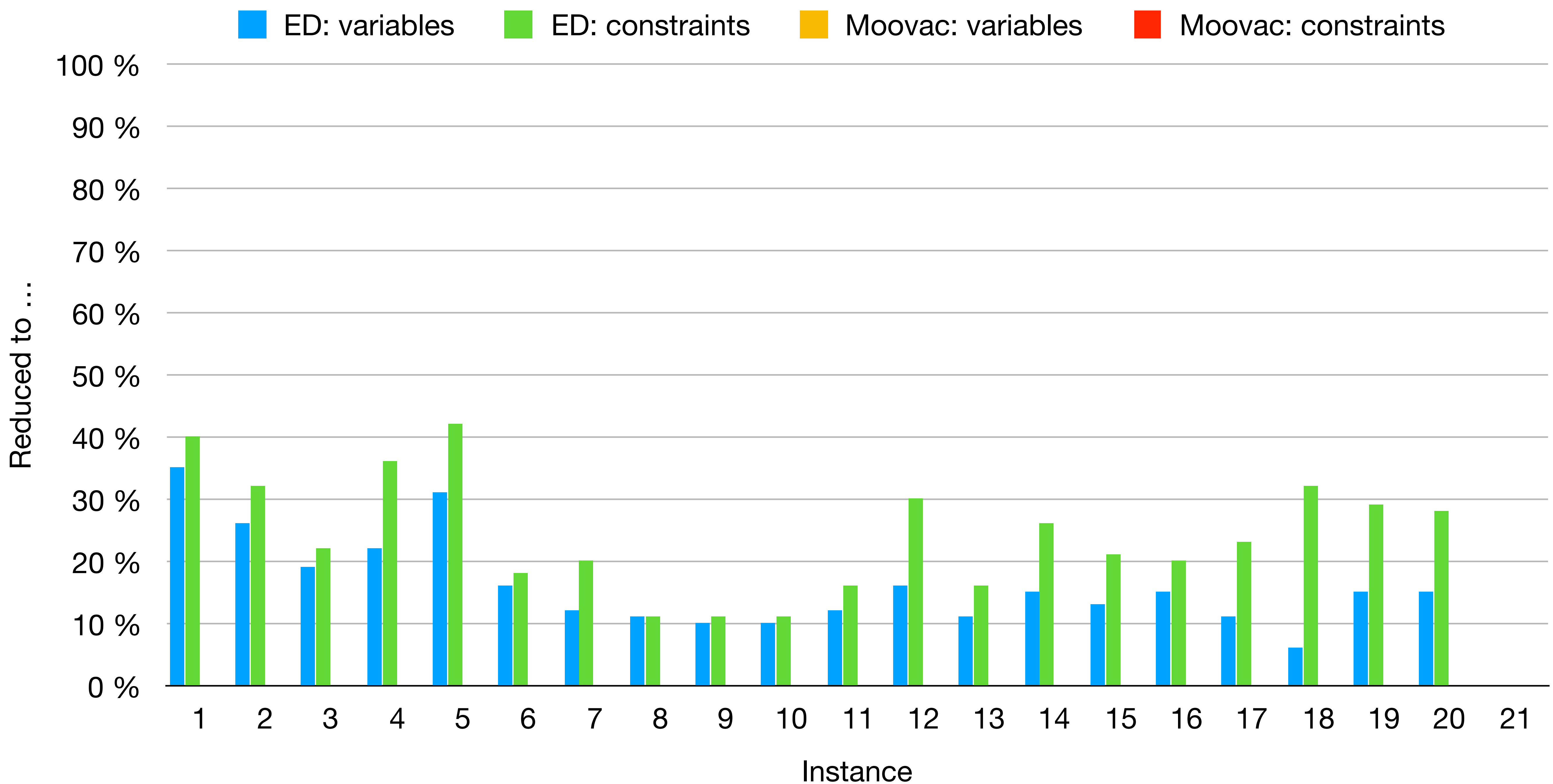

Mean #	ED	ED (red.)
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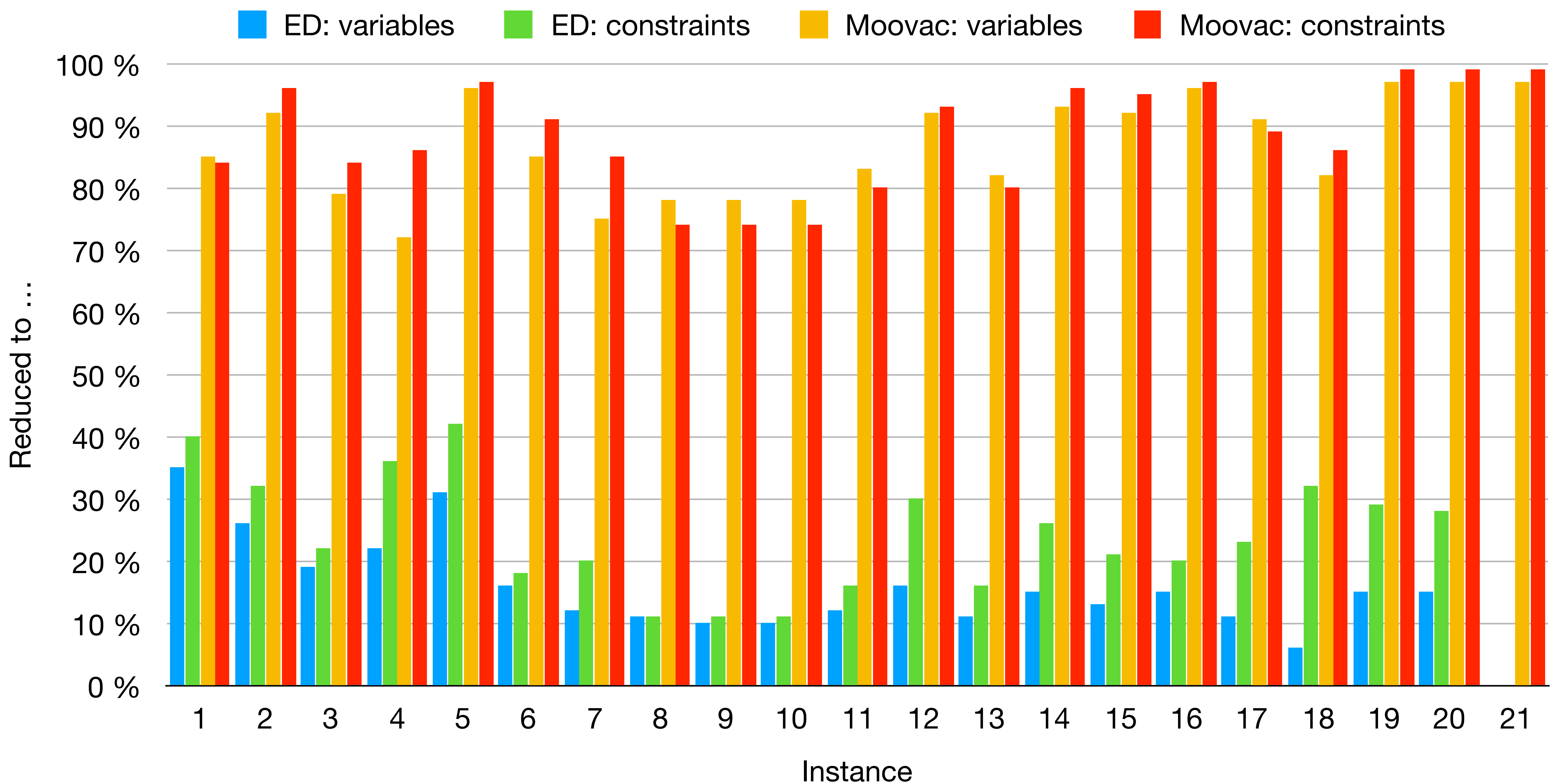

Mean #	ED	ED (red.)	Moovac	Moovac (red.)
Variables	37.2 k	5.1 k	8.4 k	7.9 k
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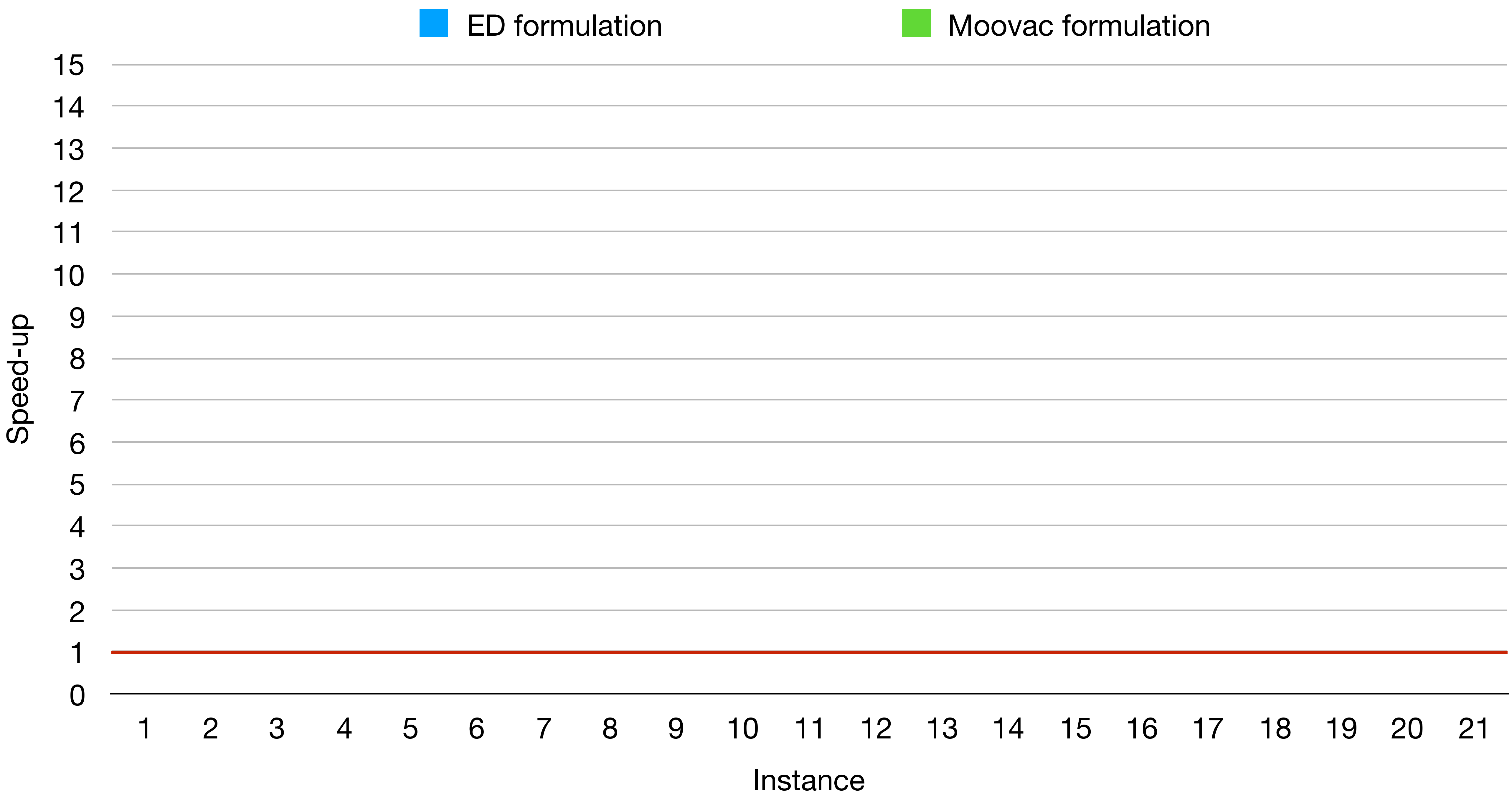
Results: Runtime



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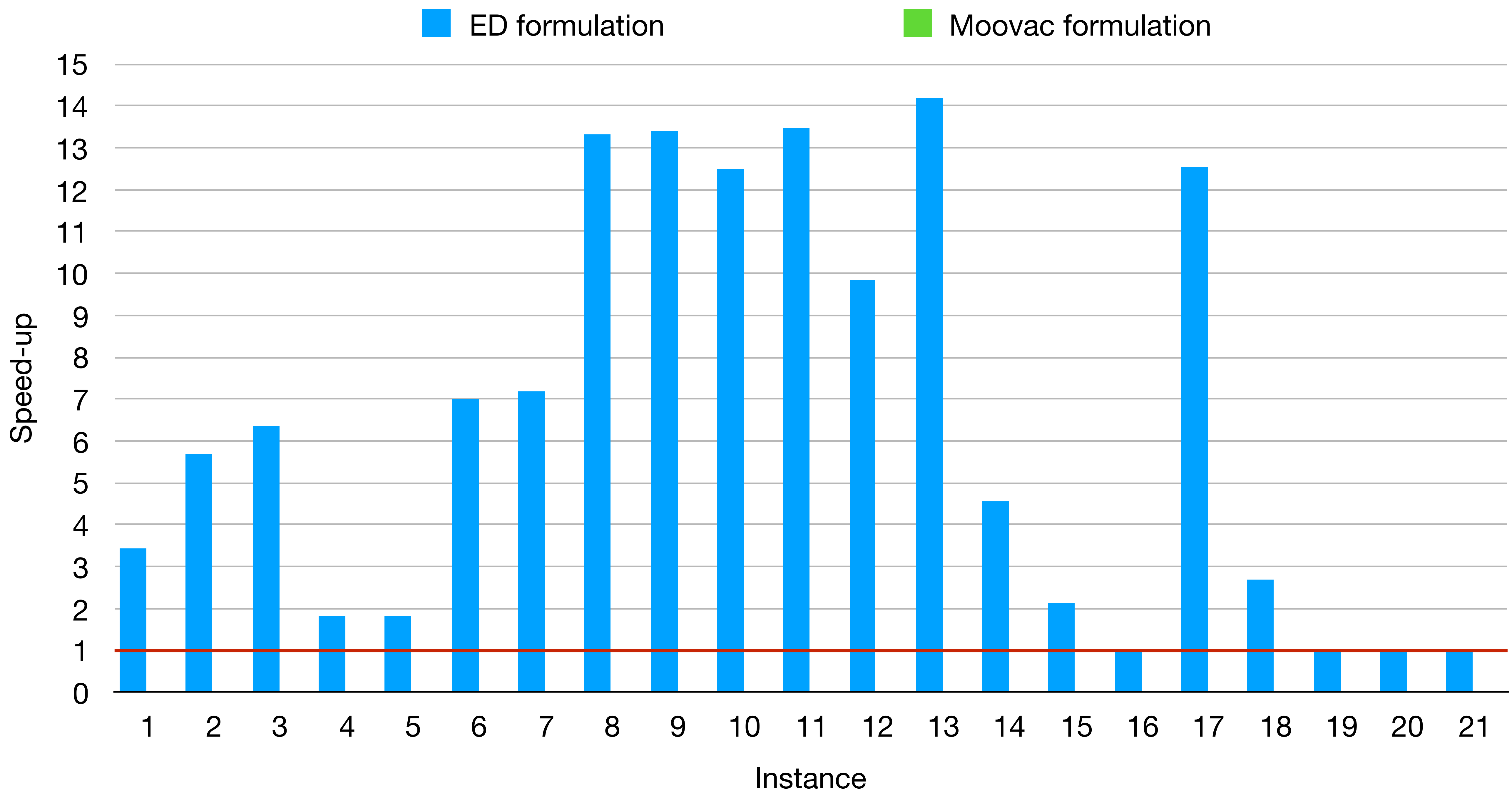

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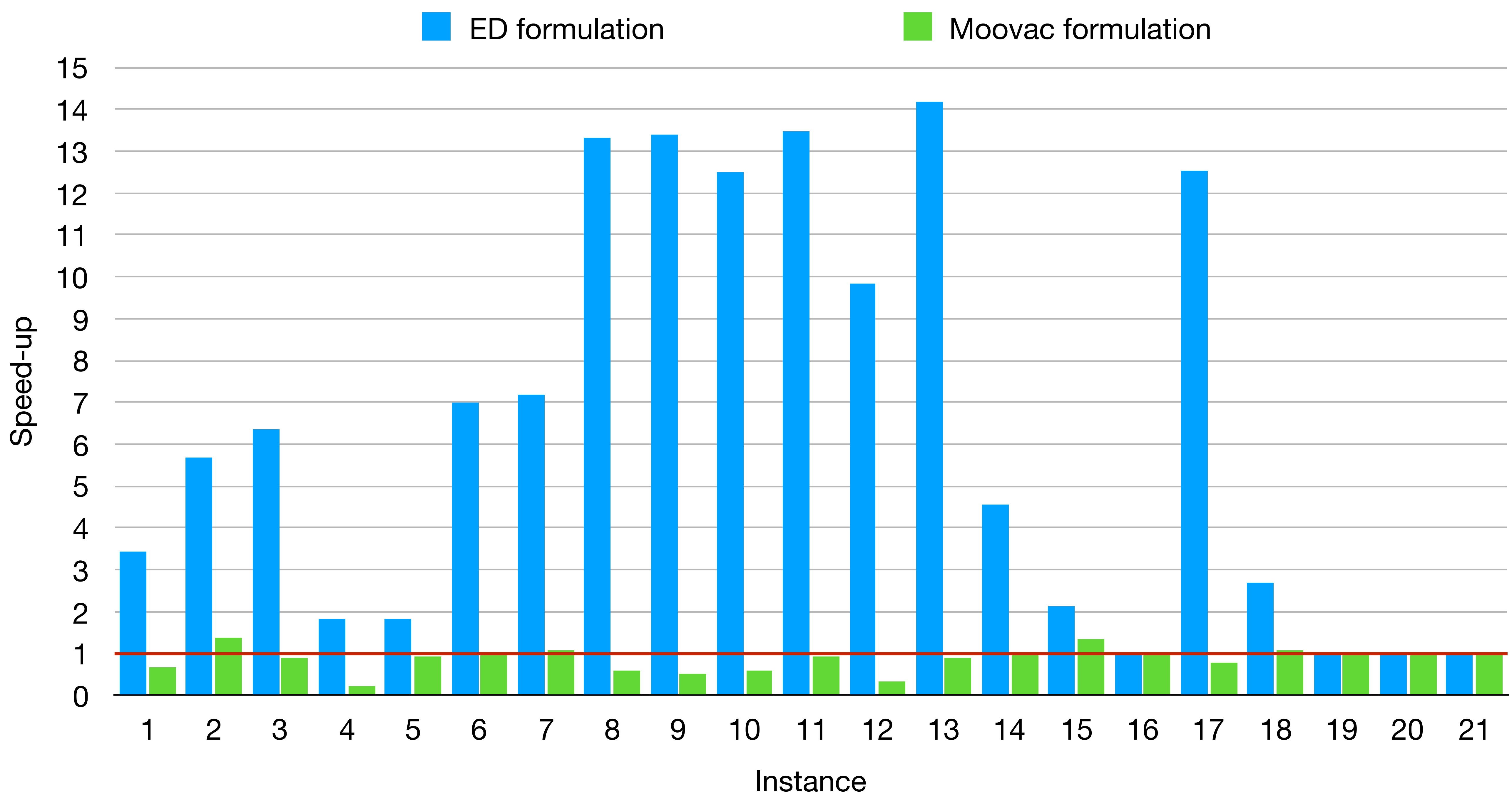

	ED	ED (red.)	Moovac	Moovac (red.)
Accumulated runtime	328 min	268 min	290 min	285 min
Speed-up (geomean)		4.37x		0.8x

Results: Solution Quality

# instances	ED	ED (red.)	Moovac	Moovac (red.)
optimal	11	14	13	12
optimal II	2	-	2	3
feasible	3	4	3	3
no solution	5	3	3	3

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- Again, minor regression for Moovac

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  - not enough reduction potential to offset ILP solvers' *„performance variability"*

- Both now **much closer** performance-wise
  - seem to **complement** each other

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- Long-term goal: an **oracle**
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  → important to have different schedulers that scale roughly the same

Thank you!

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